

# Fuzzified Job Shop Scheduling Algorithm

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## Abstract

In the present scenario the recent engineering and industrial built-up units are facing assortment of problems in a lot of aspects such as machining time, electricity, man power, raw material and customer's constraints. The Job-shop scheduling is one of the most significant industrial behaviours, particularly in manufacturing planning. Consider a shop ground where numbers of jobs are processed by number of machines. All the jobs consist of a certain number of operations. Each one operation has to be performed by a committed machine with the predefined processing time. The sequence of operations is also predefined for each job in a production recipe. Therefore, each job has its own machine order and can't change the sequence of operations. This paper introduced a new job shop scheduling algorithm with same fuzzy rules. The scope and purpose of this paper is to examine the near optimal solution for job shop scheduling in such a way that two operations are not at all processed on the same machine at the same time.

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## Keywords

Job shop scheduling, Fuzzy logic, Makespan, Flow shop scheduling.

## Introduction

Scheduling is the assignment of some shared resources with time to contesting processes. It is one of the interesting subjects in the operations research field for research. Insistence has been on finding machine scheduling problems where jobs serve for processes and machines serve for resources; one machine can perform exactly one job at an instance [9]. The Job shop scheduling problem is worst member in the class of NP-hard problem. It can be proved by fact that a  $10 * 10$  problem given by Muth and Thompson [10] remained unsolved for more than 20 years. Likewise exhaustive search methods depending on branch and bound algorithms, some approximation algorithms have been proposed. One of the most popular practices is based on priority rules and generation of active schedule [12]. Job shop scheduling problem is a problem that commonly occurs in real world application environments [8]. In a flow shop scheduling, all jobs are processed by a sequence of machines in a predefined order. The objective of a flow-shop sequencing problem is to find the sequence of jobs that minimizes the maximum flow time. Johnson published the first paper on the flow-shop problem in 1954. This problem has held the attention of many researchers and has been extensively studied in the literature [3, 4]. A survey paper concerning the flow-shop problem was presented by Dudek et al. [5]. A book has recently been published on this problem by Morton and Pentico [6]. The shifting bottleneck in [13] proposed an approximation algorithm for solving the job shop scheduling problem with minimum makespan. It identifies the bottlenecked machine among the non sequenced machines and one by one put them in sequence. It locally optimizes previously sequences whenever a new machine sequenced.

J.F. Goncalves and M.G.C. Resende [14] proposed a local search method for the job-shop scheduling problem based on a new neighborhood, and it generate schedule decoding the chromosome supplied by the genetic algorithm. A Scatter Search (SS) based algorithm presented by [15] to solve job shop scheduling problems. It considers the availability constraints in a fuzzy job shop scheduling problem. It considers that a machine can be unavailable for some reason like maintenance, repairing or due to sudden breakdown. It minimizes the tardiness and earliness in fuzzy job shop scheduling problem. S. Sundar, P. N. Suganthan, and T. J. Chua [16] proposed a flexible job-shop scheduling problem with

no-wait constraint (FJSPNW). It combines characteristics of two different job shop scheduling problems, flexible and no-wait job-shop scheduling problem. R. Zhang, S. Song, and C. Wu [17] proposed a hybrid differential evolution (DE) method to solve the job shop scheduling problem with random processing times with the target of minimizing the expected total tardiness. X Zhang, Y Deng, FTS Chan and P Xu [18] addressed proposed an intuitionistic fuzzy set Job Shop Scheduling Problem.

The rest of the paper is organized as follow: section 2 discusses job shop scheduling problem. Next section discusses fuzzy logic control system. Section 4 provides a brief overview of Gupta's heuristics for job shop scheduling. Next section proposes a new strategy for job shop scheduling followed by results and discussion.

## Job Shop Scheduling

Job shop scheduling problem is one of the popular optimization problem in engineering and operations research. In this problem ideal jobs are placed on appropriate resources at certain times. In job shop scheduling  $n$  jobs ( let  $J_1, J_2, \dots, J_n$  of different sizes) are given, which require to be scheduled on  $m$  machines, while keeping in mind that we need to minimize the makespan, minimize total tardiness, maximize utilization system/resource, minimize in-process inventory, balance usage of resource and maximize rate of production. The makespan is the total schedule length. There are a numbers of variant of the job shop scheduling studied classified as per different parameters.

- Machines may be independent from each other, related in some context, equal.
- Machines may require some gap between successive jobs or no gap.
- Machines may depend on some sequence.
- Objective function may be multi-objective optimization problem
- Jobs may have some type of constraints.
- Jobs and machines may have some mutual constraints.
- Processing time may be fixed or probabilistic.

One another similar problem is flow shop scheduling, an instance of job shop scheduling. In case of flow shop scheduling it follows strict order of all operations to be performed on all jobs. Flow shop scheduling applied to a range of problems including production facilities and computing designs. Scheduling means allocation of shared resources or machines over time to competing jobs or activities; each machine or resource can process one job at a time. Scheduling is a long lasting process where the existence of real time information repeatedly forces the appraisal and modification of pre-established schedules.

Job shop is a work location where a number of general purpose work stations or machines exist and are used to perform a variety of jobs.

Job shop scheduling is also known as job shop problem. It is an optimization predicament in which resources are allocated to ideal jobs at particular times. State  $n$  jobs  $J_1, J_2, \dots, J_n$  and each job consist a chain of operations with different sizes, which scheduled on  $m$  indistinguishable machines ( $m > 2$ ), while demanding to minimize the makespan. Each machine can process at most one operation at a time. In any process the preemption of an operation is not acceptable.

Flow shop scheduling is a special case of job shop scheduling in which all operations are having strict order to be performed all jobs. Flow shop scheduling problems are scheduling problems in which the flow control shall enable an appropriate sequencing for each job and for processing on a set of machines or with other resources  $1, 2, \dots, m$  in acquiescence with given execution orders. It maintains the constant flow of processing jobs is preferred with a minimum of idle time and a minimum of waiting time.

## Fuzzy Logic Control Systems

Fuzzy system is basically handled the numerical data or mathematical form and linguistic knowledge simultaneously. The objective knowledge of mathematical form is used in engineering problems and the subjective knowledge of linguistic form is usually impracticable to quantify. Fuzzy systems have been simulated, modelled and replicated to many real world problems. Any system may be viewed as fuzzy system which uses fuzzy mathematics. Fuzzy systems include both fuzzy logic and fuzzy set theory. Fuzzy set theory is introduced by Zadeh in 1965 [7]. Fuzzy set theory is an extension of traditional set theory wherever elements have different degrees of membership. Fuzzy set is allows its members

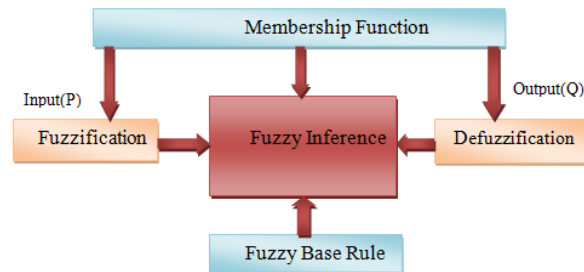
to have different degree of membership, known as membership function, with the interval between 0 and 1.

Fuzzy logic is introduced by Zadeh in 1965 [7]. Fuzzy logic is a probabilistic logic which means form of multi-valued logic; it deals with logic that is estimated rather than predetermined and exact. In distinction with conventional logic they can have changeable values, where binary sets have 2-valued logic, either true or false, whereas fuzzy logic variables may have a truth value that may ranges in degree between 0 and 1. Fuzzy logic has been extended to knob the perception of truth to a limited extent, where the truth value may span between completely true and completely false.

Application of fuzzy logic varies from decision making process to signal processing or data analysis. H. Ramazi, and A. Amini [19] applied fuzzy logic in compiling multi geohazards macro-zone maps. L.A. Zadeh implemented it to find flexibility of protein motifs [20]. Y. Huang and Y. Li use fuzzy logic to outline differences between various poly nucleotides [21]. Z. Xiu-fen, P. Zi-shu, K. Le-shan and Z. Chu-yu Analyzed data for experimental expression [22] using fuzzy theory of adaptive resonance [20]. Some other researches applied fuzzy logic in aligning sequences for dynamic programming [23, 24], generating DNA sequencing [20], identifying the cluster genes from micro-array data [25]. YQ Ren, XG Duan, HX Li, CLP Chen [26] proposed a multi-variable fuzzy logic control system for a class of distributed parameter systems. I. Pan and S. Das [27] enhanced PID Controller. M. Turkkan and N. Yagiz [28] developed active bus suspension system with help of fuzzy logic control system. A.M. Eltamaly and H. M. Farh [29] used fuzzy logic control to maximize power extraction from wind energy system. M. Togai and H. Watanabe [30] implemented for real time approximate reasoning. R. Kumari, V. K. Sharma and S. Kumar applied fuzzy logic control in air conditioning system [31], ducting system [32] and CPU scheduling [33]. A.I. Saleh [34] proposed a fuzzy matchmaking based system-oriented grid scheduler.

## Gupta Heuristic Algorithm

Gupta proposed a heuristic technique to achieve an almost minimum makespan. In the Gupta heuristic algorithm all the jobs are divided into two groups by comparing the dispensation times of the first machine and the last machine in each job. For every group, calculate the sum of processing times of any two adjacent tasks in a job and find the minimum processing time, and then schedules



**Figure 1:** Block Diagram of Fuzzy Control System

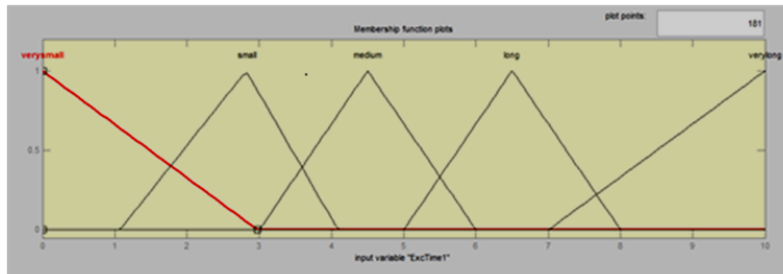
the jobs in sorting order according to their minimum summed processing times.

Johnson’s algorithm is basically used for only two machines [2], but the idea of Gupta algorithm is applicable for more than two machines [1]. This algorithm state an m machines, a set of n independent jobs with a chain of operations that must be executed in the same sequence on each machine. Gupta proposed the following heuristic algorithm to solve it in polynomial time [1]. Algorithm 1 outline Gupta’s heuristic.

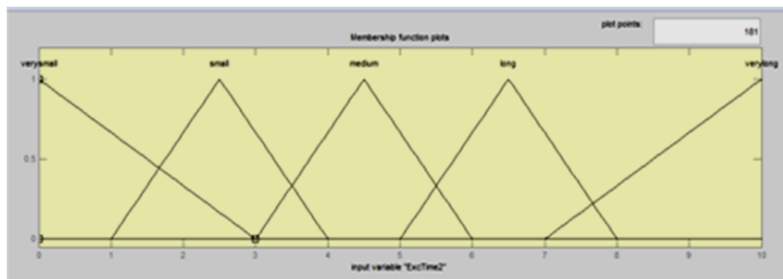
## Proposed Algorithm

The proposed job shop scheduling algorithm is a variant of gupta’s heuristic. This algorithm is combination of Guptas heuristic and Johnson’s algorithm. It deals with some fuzzy logic rules and these rules are based on each operation time of a job. It calculates a new value of execution with the help of operation time of each job for combination of two machines (like, machine 1 and machine 2, machine 2 and machine 3 and so on). This paper use suitable linguistic variables as input and output to compute a crisp value for new execution time. Execution Time1 (ET1), Execution Time2 (ET2) and New Execution Time (NET) are measured as very small, small, medium, long and very long. The proposed job shop scheduling algorithm is a collection of linguistic fuzzy rules which describe the relation between defined input variables (ET1 and ET2) and output (NET). Algorithm 2 outlines steps of proposed algorithm.

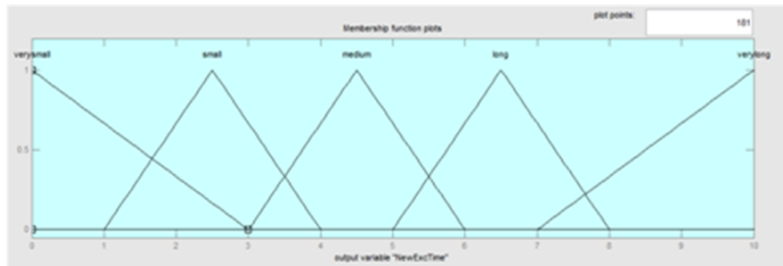
Figure 2 and 3 shows plot of membership functions for input variables Execution Time 1 and Execution Time 2. Figure 4 shows plot of membership functions for output variable New Execution Time 3.



**Figure 2:** Membership Function for Input Variable Execution Time 1



**Figure 3:** Membership Function for Input Variable Execution Time 2



**Figure 4:** Membership Function for Input Variable Execution Time 3

Table 1 shows membership functions and ranges for input and output variables Execution Time 1, Execution Time 2 and New Execution Time.

This algorithm used 25 rules which are based on IF THEN statement such as: - IF ET1 is Very small and ET2 is Very Small THEN NET is Very Small.

| Execution Time 1 | Execution Time 2 | New Execution Time 3 | Range  |
|------------------|------------------|----------------------|--------|
| Very Small       | Very Small       | Very Small           | 0 - 3  |
| Small            | Small            | Small                | 1 - 4  |
| Medium           | Medium           | Medium               | 3 - 6  |
| Long             | Long             | Long                 | 5 - 8  |
| Very Long        | Very Long        | Very Long            | 7 - 10 |

**Table 1:** Membership functions and ranges for input and output variables

These rules are outlined in table 2.

| Execution Time 1 | Execution Time 2 | New Execution Time 3 |
|------------------|------------------|----------------------|
| Very Small       | Very Small       | Very Small           |
| Very Small       | Small            | Very Small           |
| Very Small       | Medium           | Small                |
| Very Small       | Long             | Medium               |
| Very Small       | Very Long        | Long                 |
| Small            | Very Small       | Very Small           |
| Small            | Small            | Small                |
| Small            | Medium           | Small                |
| Small            | Long             | Medium               |
| Small            | Very Long        | Long                 |
| Medium           | Very Small       | Small                |
| Medium           | Small            | Small                |
| Medium           | Medium           | Medium               |
| Medium           | Long             | Long                 |
| Medium           | Very Long        | Very Long            |
| Long             | Very Small       | Medium               |
| Long             | Small            | Medium               |
| Long             | Medium           | Long                 |
| Long             | Long             | Long                 |
| Long             | Very Long        | Very Long            |
| Very Long        | Very Small       | Long                 |
| Very Long        | Small            | Long                 |
| Very Long        | Medium           | Very Long            |
| Very Long        | Long             | Very Long            |
| Very Long        | Very Long        | Very Long            |

**Table 2:** Set of proposed rules



## Experimental Results and Discussion

This section compare gupta's heuristic and proposed method with the help of an example. In this example it is assumed that there are five jobs and three machines with different execution time on each machine in particular order.

Example: Assume five jobs  $J1$  to  $J5$  are to be scheduled. Each job has executed on three machines ( $M1, M2$  and  $M3$ ). The execution time for each task is as shown in Table 3.

| Job Machines | Machine M1 | Machine M2 | Machine M3 |
|--------------|------------|------------|------------|
| Job J1       | 4          | 7          | 3          |
| Job J2       | 1          | 5          | 2          |
| Job J3       | 5          | 2          | 4          |
| Job J4       | 2          | 5          | 2          |
| Job J5       | 5          | 5          | 6          |

Table 3: Processing time of five jobs on three machines

### The Gupta scheduling algorithm, the execution process for above example is as follows

- Step 1: The group P is having the job as J2, J4, J5.
- Step 2: The group Q is having the job as J1, J3.
- Step 3: Find the minimum value for each job in group P  
 $\min\{1+5, 5+2\} = \min\{6, 7\} = 6,$   
 $\min\{2+5, 5+3\} = \min\{7, 8\} = 7,$   
 $\min\{5+5, 5+6\} = \min\{10, 11\} = 10.$
- Step 4: Find the minimum value for each job in group Q  
 $\min\{4+7, 7+3\} = \min\{11, 10\} = 10,$   
 $\min\{5+2, 2+4\} = \min\{7, 6\} = 6.$
- Step 5: Sort the jobs in P as {J2, J4, J5}.
- Step 6: Sort the jobs in Q as {J1, J3}.
- Step 7: The complete sequence is J2, J4, J5, J1, J3.

Now schedule the tasks on the machines and the final scheduling result is shown in figure 5. The calculated makespan is 30.

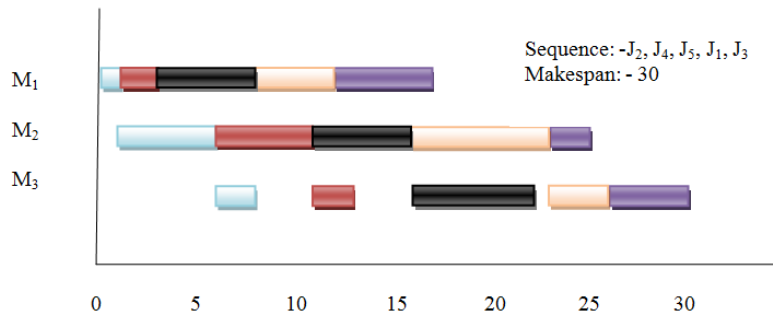


Figure 5: Schedule for Jobs  $J_2, J_4, J_5, J_1, J_3$ , as per Gupta's Heuristic

### The proposed algorithm the execution of processes is as follows

- Step 1: Calculate the centroid by using fuzzy logic and centroid is shown in table ??.
- Step 2: The group X is having the jobs as  $\{J_2, J_4, J_5\}$ .
- Step 3: The group Y is having the jobs as  $\{J_1, J_3\}$ .
- Step 4: Ordered group is as X  $\{J_2, J_4, J_5\}$  and Y  $\{J_3, J_1\}$ .
- Step 5: Take all the jobs of group Y  $\{J_3, J_1\}$  and find the minimize partial makespan. Makespan of sequence J1, J3 is 18 (figure 6) and sequence J3, J1 is 19 (figure 7), so selected sequence is J1, J3.
- Step 6: Now insert all the job of group X one by one and select minimized makespan sequence at every time. Makespan of sequence J2, J1, J3 is 20, (figure 8) sequence J1, J2, J3 is 22 (figure 9) and sequence J1, J3, J2 is 20 (figure 10) so selected sequences are J2, J1, J3 and J1, J3, J2. Makespan of sequence J4, J2, J1, J3 is 26 (figure 11), sequence J2, J4, J1, J3 is 25 (figure 12), sequence J2, J1, J4, J3 is 24 (figure 13), sequence J2, J1, J3, J4 is 22 (figure 14), sequence J4, J1, J3, J2 is 23 (figure 15), sequence J1, J4, J3, J2 (figure 16), sequence J1, J3, J4, J2 (figure 17) and sequence J1, J3, J2, J4 is 25 (figure 18).  
Selected sequences are J2, J1, J3, J4 and J4, J1, J3, J2.  
Thus the best sequence is J2, J5, J1, J3, J4 with makespan 27 and other good sequences are J4, J5, J1, J3, J2 and J2, J1, J3, J5, J4 both with makespan of 28. Complete results are outlined in table 4.

Makespan of sequence J4, J2, J1, J3 is 26, sequence J2, J4, J1, J3 is 25, sequence J2, J1, J4, J3 is 24, sequence J2, J1, J3, J4 is 22, sequence J4, J1, J3, J2 is 23, sequence J1, J4, J3, J2, sequence J1, J3, J4, J2 and sequence J1, J3, J2, J4 is 25. Selected sequences are J2, J1, J3, J4 and J4, J1, J3, J2.

| Algorithm                               | Best Job Sequence  | Makespan |
|---|--------------------|----------|
| Gupta's Heuristics                      | J2, J4, J5, J1, J3 | 30       |
| Fuzzified Job Shop Scheduling Algorithm | J2, J5, J1, J3, J4 | 27       |
|   | J2, J1, J3, J5, J4 | 28       |
|   | J4, J5, J1, J3, J2 | 28       |
|   | J4, J1, J3, J5, J2 | 29       |
|   | J2, J1, J5, J3, J4 | 30       |

Table 4: Results for example by Gupta's heuristic and Fuzzified Job Shop Scheduling

## Conclusion

This paper proposed a fuzzified job shop scheduling algorithm for more than two machines. This algorithm provides far better result than Gupta's heuristic. It provides minimum final makespan as well as better partial makespan. Proposed algorithm also provides more than one choice for optimum result. As table 5 show that there are four better sequences with makespan less then Gupta's heuristic and one sequence has equal makespan. So, the proposed fuzzified job shop scheduling algorithm able to reduce the makespan up to 10

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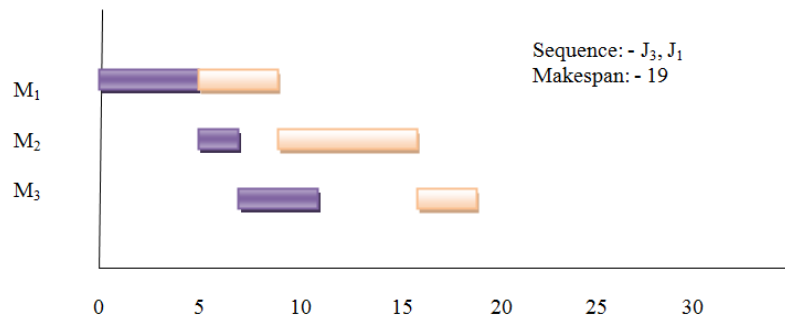
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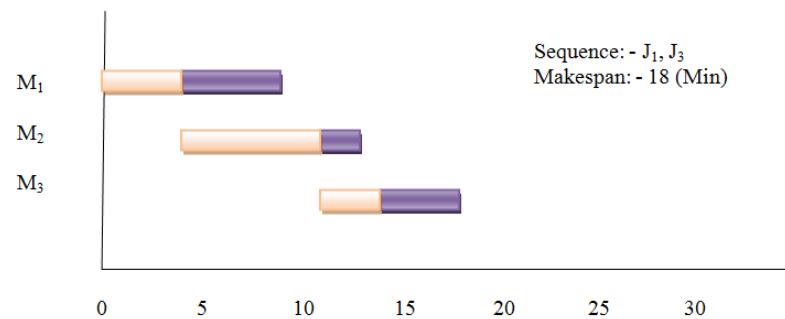
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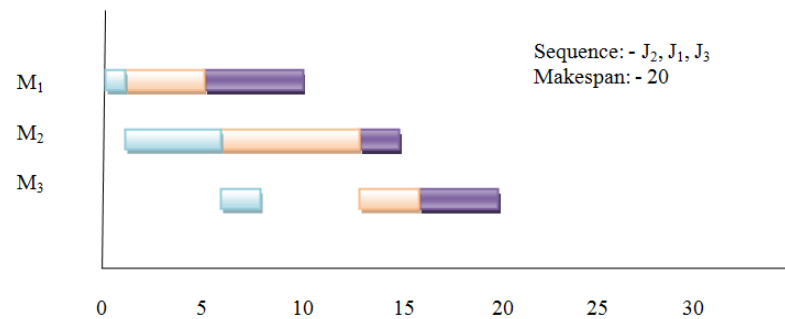
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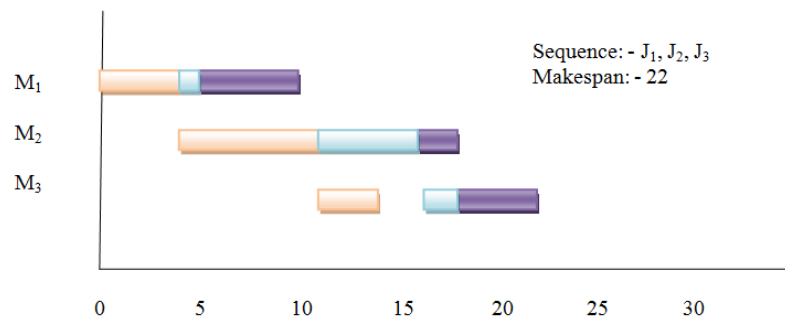
**Figure 6:** Schedule for Jobs of group Y: J<sub>3</sub>, J<sub>1</sub> according to proposed algorithm



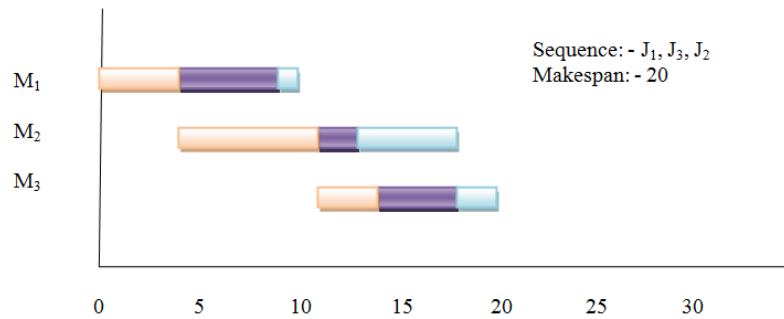
**Figure 7:** Schedule for Jobs of group Y: J<sub>1</sub>, J<sub>3</sub> according to proposed algorithm



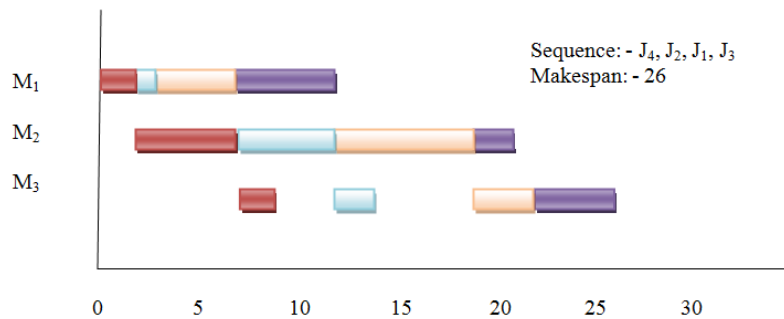
**Figure 8:** Schedule for Jobs of group X: J<sub>2</sub>, J<sub>1</sub>, J<sub>3</sub> according to proposed algorithm



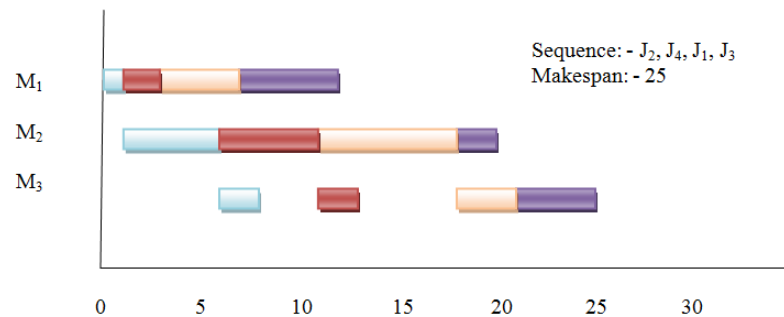
**Figure 9:** Schedule for Jobs of group X: J<sub>1</sub>, J<sub>2</sub>, J<sub>3</sub> according to proposed algorithm



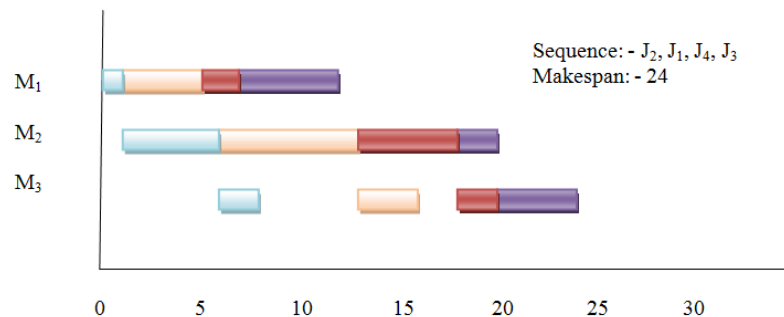
**Figure 10:** Schedule for Jobs of group X:  $J_1, J_3, J_2$  according to proposed algorithm



**Figure 11:** Schedule for Jobs:  $J_4, J_2, J_1, J_3$  according to proposed algorithm

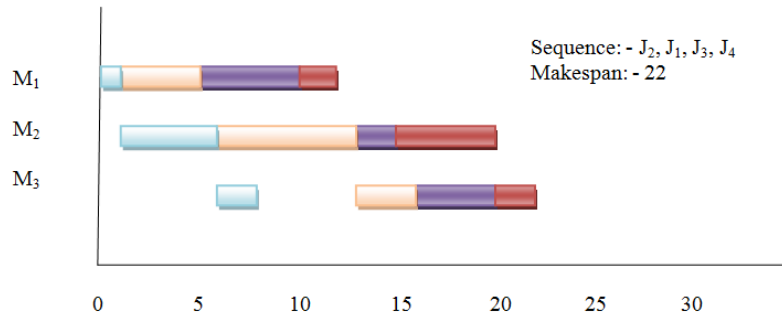


**Figure 12:** Schedule for Jobs:  $J_2, J_4, J_1, J_3$  according to proposed algorithm

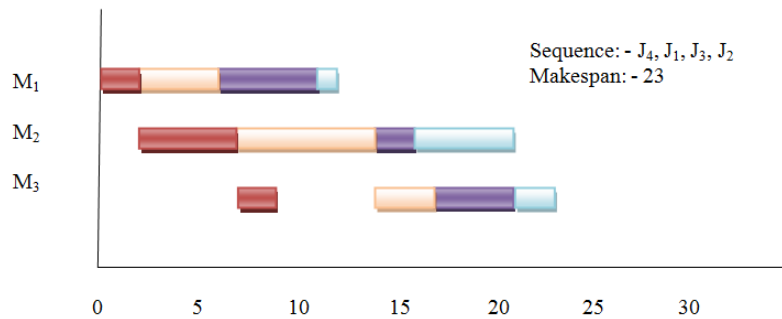


**Figure 13:** Schedule for Jobs:  $J_2, J_1, J_4, J_3$  according to proposed algorithm

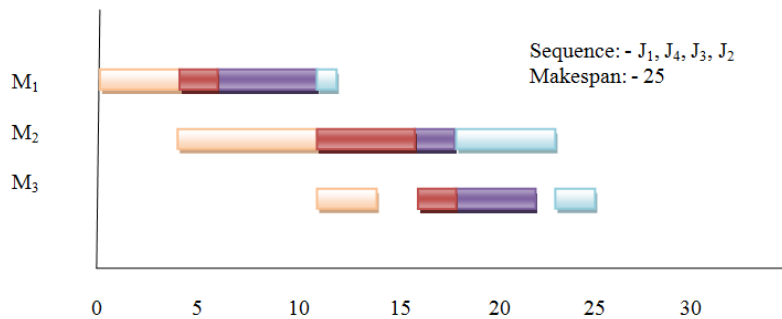




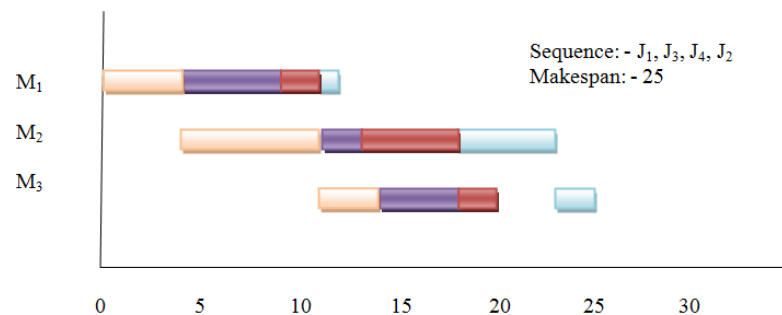
**Figure 14:** Schedule for Jobs: J<sub>2</sub>, J<sub>1</sub>, J<sub>3</sub>, J<sub>4</sub> according to proposed algorithm



**Figure 15:** Schedule for Jobs: J<sub>4</sub>, J<sub>1</sub>, J<sub>3</sub>, J<sub>2</sub> according to proposed algorithm



**Figure 16:** Schedule for Jobs: J<sub>1</sub>, J<sub>4</sub>, J<sub>3</sub>, J<sub>2</sub> according to proposed algorithm



**Figure 17:** Schedule for Jobs: J<sub>1</sub>, J<sub>3</sub>, J<sub>4</sub>, J<sub>2</sub> according to proposed algorithm

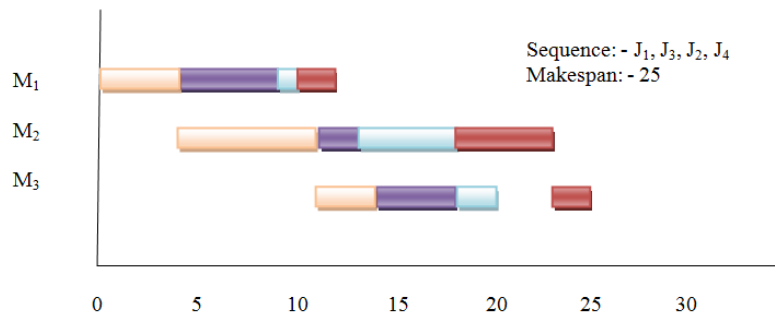


Figure 18: Schedule for Jobs:  $J_1, J_3, J_2, J_4$  according to proposed algorithm

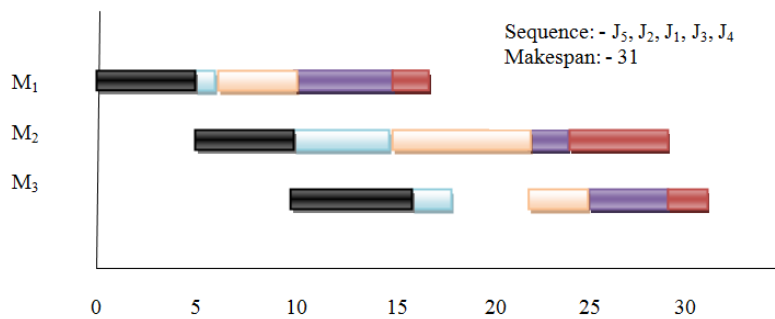


Figure 19: Schedule for Jobs:  $J_5, J_2, J_1, J_3, J_4$  according to proposed algorithm

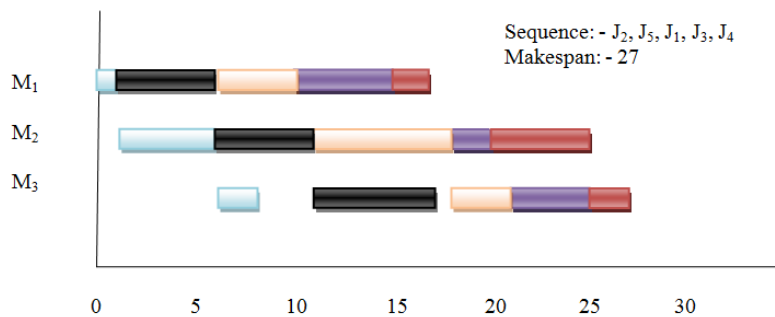


Figure 20: Schedule for Jobs:  $J_2, J_5, J_1, J_3, J_4$  according to proposed algorithm

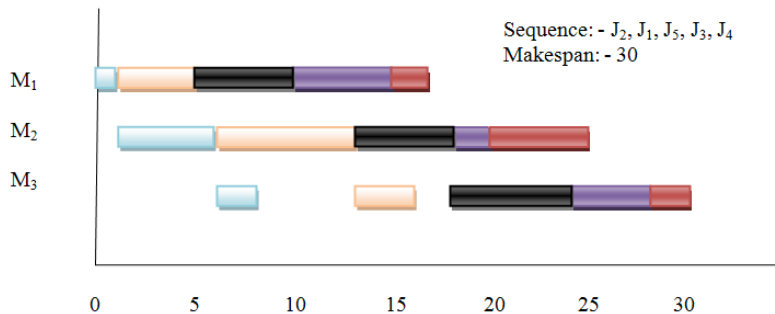
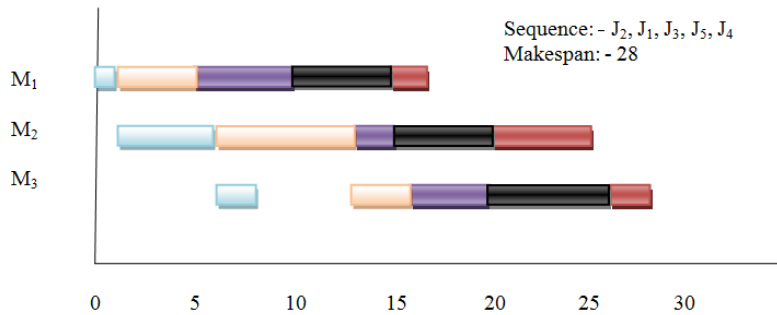
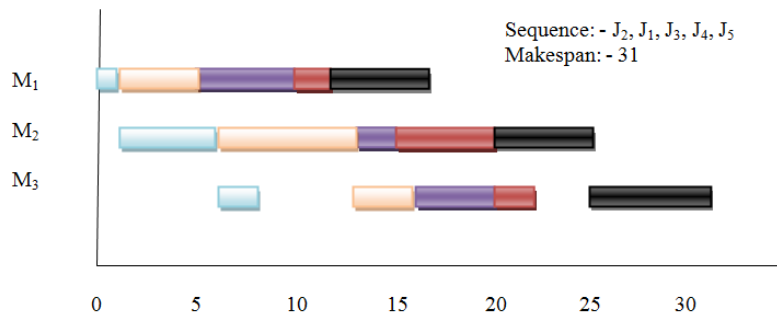


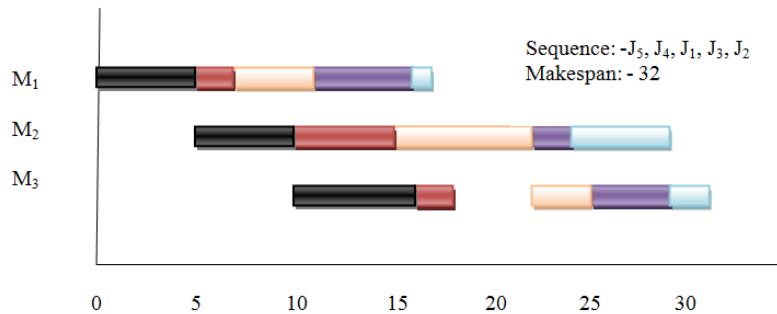
Figure 21: Schedule for Jobs:  $J_2, J_1, J_5, J_3, J_4$  according to proposed algorithm



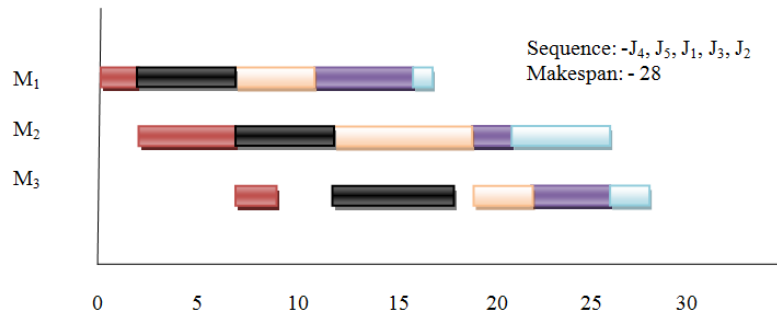
**Figure 22:** Schedule for Jobs:  $J_2, J_1, J_3, J_5, J_4$  according to proposed algorithm



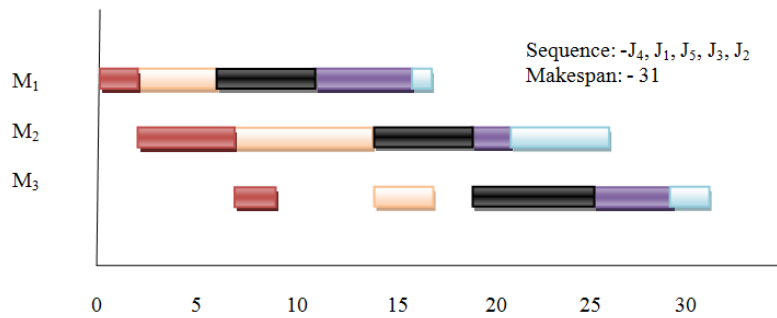
**Figure 23:** Schedule for Jobs:  $J_2, J_1, J_3, J_4, J_5$  according to proposed algorithm



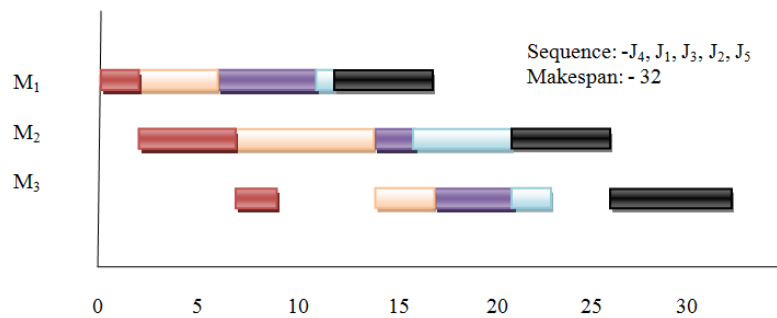
**Figure 24:** Schedule for Jobs:  $J_5, J_4, J_1, J_3, J_2$  according to proposed algorithm



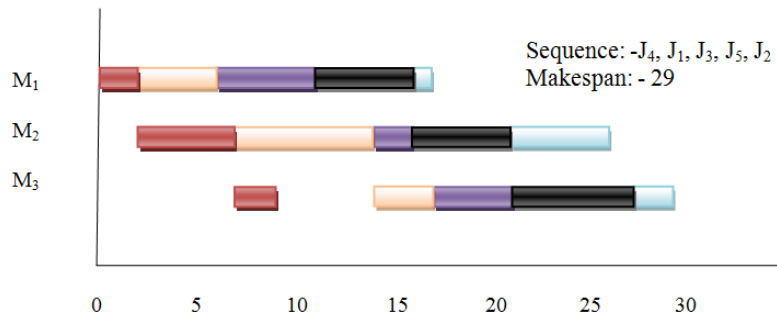
**Figure 25:** Schedule for Jobs:  $J_4, J_5, J_1, J_3, J_2$  according to proposed algorithm  
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Fuzzified Job Shop Scheduling Algorithm.



**Figure 26:** Schedule for Jobs:  $J_4, J_1, J_5, J_3, J_2$  according to proposed algorithm



**Figure 27:** Schedule for Jobs:  $J_4, J_1, J_3, J_2, J_5$  according to proposed algorithm



**Figure 28:** Schedule for Jobs:  $J_4, J_1, J_3, J_5, J_2$  according to proposed algorithm