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LMI Approach to Optimal Guaranteed Cost Control for Uncertain 2-D Discrete Shift Delayed Systems Described by the General Model

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Abstract

This paper is related to the robust stability analysis of 2-D discrete systems, subjected to norm bounded uncertainties as described by the shift delayed General model. A new criterion is developed for robust optimal guaranteed cost control of 2-D discrete systems via memory state feedback. An LMI (linear matrix inequality) based convex optimization problem is devised to design optimal controllers that stabilizes the underline system as well as provides an upper bound on the performance index. Finally a thermal process example is used to illustrate the effectiveness of the proposed result.

Keywords

2-D discrete systems, shift-delay, robust stability, memory state feedback, guaranteed cost control.

Introduction

2-D discrete systems have received considerable attention due to their existence and extensive applications in modern engineering areas, for e.g. thermal processes, image processing, signal filtering etc. [1-5]. The underline feature of 2-D system in gaining so much popularity is that the information flow takes place in two independent directions. A

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generalized state-space model, called General Model, to represent the 2-D discrete systems is proposed in [6]. A survey presented in [7, 8] shows important results for 2-D discrete systems described by the other popular state-space models. Robust stability [9] and performance analysis of 2-D discrete systems is one among the most sought aspect in the on-going research. The guaranteed cost control approach [10] not only robustly stabilizes the system, but also guarantees an adequate level of performance of the system subjected to uncertainties. Based upon this approach, many important results have been reported in literature [10-15, 17-20, 23] for uncertain 2-D discrete systems.

The subject of uncertain 2-D discrete shift- delayed systems has also received increasing attention [16-25] in recent past since not only the parametric uncertainty, but also the shift delays are the main cause of instability and subsequently the performance deterioration of any system. Shift-delays appear due to the spreading process that occurs during information transmission or during computation time along time and space. Robust stability and stabilization for a class of 2-D discrete shift-delayed systems is considered in [16]. The problem of stability of 2-D discrete shift-delayed systems described by the General model is studied in [21, 22]. The problem of robust H_∞ control for 2-D discrete shift-delayed systems is explored in [24, 25]. Further, the issue of guaranteed cost control for a class of uncertain 2-D discrete shift-delayed system via memory less state-feedback is dealt in [17, 18] and via memory state feedback is studied in [19]. It may be noted that 2-D discrete systems described by the General model, which is more versatile than some of the other state space models, has not been addressed by many researchers. In particular, a 2-D discrete shift-delayed system described by the General model has made rare appearance in the literature [21, 22]. Motivated by this we take up the problem of optimal guaranteed cost control via memory state feedback for 2-D discrete shift delayed systems, described by the General model. To the best of author's knowledge such problem has not been reported so far in the literature.

This paper is organized as follows: Robust guaranteed cost control problem for an uncertain 2-D discrete shift-delayed General model is formulated in Section 2. Some important results are also recalled. An LMI approach for the optimal guaranteed cost control via memory state feedback is presented in Section 3. In Section 4 a numerical example based upon the thermal process with both space and time delays is provided to illustrate the presented technique.

Notations: Throughout this paper the following notations are used: matrix transposition is represented by superscript T, R^n denotes the real vector space of dimension n, the set of $n \times m$ real matrices is represented by $R^{n\times m}$, null matrix or null vector of appropriate dimension is represented by 0, identity matrix of appropriate dimension is represented by $\boldsymbol{I}$, * denotes symmetry , det (.) represents determinant of a matrix, and diagonal {….} represents a block diagonal matrix. Moreover, a matrix $\boldsymbol{G}$ which is real symmetric and positive (or, negative) definite is represented by $\boldsymbol{G} > \boldsymbol{0}$ (or, $\boldsymbol{G} < \boldsymbol{0}$) and $\lambda_{\max}(\boldsymbol{G})$ stands for maximum Eigen value for the matrix $\boldsymbol{G}$.

Problem Formulation and Preliminaries

In this paper, the problem of robust optimal guaranteed cost control for a class of 2-D discrete systems described by the shift delayed General model is carried out. Specifically, the system under consideration is given by

$$\begin{aligned}\boldsymbol{x}(i+1, j+1) &= (\boldsymbol{A}_1 + \Delta\boldsymbol{A}_1)\boldsymbol{x}(i, j+1) + (\boldsymbol{A}_{1d} + \Delta\boldsymbol{A}_{1d})\boldsymbol{x}(i-d_1, j+1) + (\boldsymbol{A}_2 + \Delta\boldsymbol{A}_2)\boldsymbol{x}(i+1, j) \\ &+ (\boldsymbol{A}_{2k} + \Delta\boldsymbol{A}_{2k})\boldsymbol{x}(i+1, j-k_1) + (\boldsymbol{A}_3 + \Delta\boldsymbol{A}_3)\boldsymbol{x}(i, j) + (\boldsymbol{A}_{3dk} + \Delta\boldsymbol{A}_{3dk})\boldsymbol{x}(i-d_2, j-k_2) \\ &+ (\boldsymbol{B}_1 + \Delta\boldsymbol{B}_1)\boldsymbol{u}(i, j+1) + (\boldsymbol{B}_2 + \Delta\boldsymbol{B}_2)\boldsymbol{u}(i+1, j) + (\boldsymbol{B}_3 + \Delta\boldsymbol{B}_3)\boldsymbol{u}(i, j)\ , \qquad (1a)\end{aligned}$$

where $\boldsymbol{x}(i, j) \in R^n$ and $\boldsymbol{u}(i, j) \in R^m$ are state and control input respectively. The matrices $\boldsymbol{A}_1, \boldsymbol{A}_{1d}, \boldsymbol{A}_2, \boldsymbol{A}_{2k}, \boldsymbol{A}_3, \boldsymbol{A}_{3dk} \in R^{n\times n}$ and $\boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{B}_3 \in R^{n\times m}$ are known constant matrices representing the nominal plant d_1, d_2, k_1 and k_2 are constant positive integers representing delays along horizontal and vertical directions, respectively. The parameter uncertainties are represented by matrices $\Delta\boldsymbol{A}_1, \Delta\boldsymbol{A}_{1d}, \Delta\boldsymbol{A}_2, \Delta\boldsymbol{A}_{2k}, \Delta\boldsymbol{A}_3, \Delta\boldsymbol{A}_{3dk}$ and $\Delta\boldsymbol{B}_1, \Delta\boldsymbol{B}_2, \Delta\boldsymbol{B}_3$ which are further assumed to be of the form:

$$\Delta\boldsymbol{A} = \boldsymbol{L}\,\boldsymbol{F}(i, j)\,\boldsymbol{M}_1,\ \Delta\boldsymbol{B} = \boldsymbol{L}\,\boldsymbol{F}(i, j)\boldsymbol{M}_2\,. \qquad (1b)$$

$$\Delta\boldsymbol{A} = [\Delta\boldsymbol{A}_1\ \boldsymbol{A}_{1d}\ \Delta\boldsymbol{A}_2\ \Delta\boldsymbol{A}_{2k}\ \Delta\boldsymbol{A}_3\ \Delta\boldsymbol{A}_{3dk}],\ \Delta\boldsymbol{B} = [\Delta\boldsymbol{B}_1\ \Delta\boldsymbol{B}_2\ \Delta\boldsymbol{B}_3]\,. \qquad (1c)$$

$$\boldsymbol{M}_1 = [\boldsymbol{M}_{11}\ \boldsymbol{M}_{11d}\ \boldsymbol{M}_{12}\ \boldsymbol{M}_{12k}\ \boldsymbol{M}_{13}\ \boldsymbol{M}_{13dk}],\ \boldsymbol{M}_2 = [\boldsymbol{M}_{21}\ \boldsymbol{M}_{22}\ \boldsymbol{M}_{23}]\,. \qquad (1d)$$

In the above equations

$\boldsymbol{L} \in R^{n\times p}, \boldsymbol{M}_{11}, \boldsymbol{M}_{11d}, \boldsymbol{M}_{12}, \boldsymbol{M}_{12k}, \boldsymbol{M}_{13}, \boldsymbol{M}_{13dk} \in R^{q\times n}, \boldsymbol{M}_{21}, \boldsymbol{M}_{22}, \boldsymbol{M}_{23} \in R^{q\times m}$ can be considered as known structural matrices of uncertainty. $\boldsymbol{F}(i, j) \in R^{p\times q}$ can be considered as an unknown matrix representing parameter uncertainty satisfying $\boldsymbol{F}^T(i, j)\boldsymbol{F}(i, j) \le \boldsymbol{I}$,

(Or $\|\boldsymbol{F}(i, j)\| \le 1$ equivalently).(1e)

System (1a) is assumed to have a finite set of initial conditions [21-22] that there exist two integers $r_1 > 0$ and $r_2 > 0$ such that

$$\boldsymbol{x}(i, j) = \boldsymbol{0}\ \forall\ j \ge r_1,\ -d_1 \le i \le 0, \qquad \boldsymbol{x}(i, j) = \boldsymbol{\psi}_{ij},\ 0 \le j < r_1,\ -d_1 \le i \le 0,$$

$$x(i,j)=\mathbf{0}\ \forall\ i\ge r_2,\ -k_1\le j\le 0,\qquad x(i,j)=\varphi_{ij},\ 0\le i<r_2,\ -k_1\le j\le 0,$$

$$x(i,j)=\theta_{ij},\ -d_2\le i\le 0,\ -k_2\le j\le 0,\quad \theta_{00}=\varphi_{00}=\psi_{00}. \tag{1f}$$

It is further assumed that initial conditions are arbitrary, but belong to set

$$S=\{x(l,j)\in R^n : x(l,j)=MN_j,\ N_j^T N_j\le I, -d_1\le l\le 0,\ 0\le j<r_1\},$$
$$\cup\ \{x(i,l)\in R^n : x(i,l)=MN_i,\ N_i^T N_i\le I, -k_1\le l\le 0,\ 0\le i<r_2\},$$
$$\cup\ \{x(t,l)\in R^n : x(t,l)=MN_{tl},\ N_{tl}^T N_{tl}\le I, -d_2\le t\le 0,\ -k_2\le l\le 0\}. \tag{1g}$$

Cost function which is associated with system (1a) can be represented by

$$J=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[u^T(i,j+1)R_1u(i,j+1)+u^T(i+1,j)R_2u(i+1,j)+u^T(i,j)R_3u(i,j)\right]+\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}^T W_1\xi_{ij} \tag{2a}$$

where $\mathbf{0}<R_k=R_k^T\in R^{m\times m}\quad (k=1,2,3)$, (2b)

$$\xi_{ij}=[x^T(i,j+1)\quad x^T(i-d_1,j+1)\quad x^T(i+1,j)\quad x^T(i+1,j-k_1)\quad x^T(i,j)\quad x^T(i-d_2,j-k_2)]^T \tag{2c}$$

$$W_1=\begin{bmatrix} S_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & S_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & S_3 & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & S_4 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & S_5 & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & S_6 \end{bmatrix},\ \text{where } \mathbf{0}<S_t=S_t^T\in R^{n\times n}\quad (t=1,2,3,4,5,6). \tag{2d}$$

The aim of the present technique is to develop a memory state feedback control law if the system states are measurable and are available for feedback.

$$\begin{bmatrix} u(i,j+1)\\ u(i+1,j)\\ u(i,j)\end{bmatrix}=\begin{bmatrix} K_1 & K_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & K_3 & K_4 & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & K_5 & K_6\end{bmatrix}\begin{bmatrix} x(i,j+1)\\ x(i-d_1,j+1)\\ x(i+1,j)\\ x(i+1,j-k_1)\\ x(i,j)\\ x(i-d_2,j-k_2)\end{bmatrix}, \tag{3}$$

where K_1,K_2,K_3,K_4,K_5 and K_6 are appropriately dimensioned stabilizing control law matrices to be determined, for system (1a-1g) and the cost function (2a), such that the closed loop system is represented by

$$\begin{aligned} x(i+1,j+1)={}&[(A_1+\Delta A_1)+(B_1+\Delta B_1)K_1]x(i,j+1)+[(A_{1d}+\Delta A_{1d})+(B_1+\Delta B_1)K_2]x(i-d_1,j+1)\\ &+[(A_2+\Delta A_2)+(B_2+\Delta B_2)K_3]x(i+1,j)+[(A_{2k}+\Delta A_{2k})+(B_2+\Delta B_2)K_4]x(i+1,j-k_1)\\ &+[(A_3+\Delta A_3)+(B_3+\Delta B_3)K_5]x(i,j)+[(A_{3dk}+\Delta A_{3dk})+(B_3+\Delta B_3)K_6]x(i-d_2,j-k_2) \end{aligned} \tag{4}$$

and the cost function satisfies the bound

$$J=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\xi_{ij}^T W_2\xi_{ij}, \tag{5a}$$

here $J \le J^*$, where J^* is some specified constant and,

$$W_2 = W_1 + \begin{bmatrix} K_1^T R_1 K_1 & K_1^T R_1 K_2 & 0 & 0 & 0 & 0 \\ K_2^T R_1 K_1 & K_2^T R_1 K_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_3^T R_2 K_3 & K_3^T R_2 K_4 & 0 & 0 \\ 0 & 0 & K_4^T R_2 K_3 & K_4^T R_2 K_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_5^T R_3 K_5 & K_5^T R_3 K_6 \\ 0 & 0 & 0 & 0 & K_6^T R_3 K_5 & K_6^T R_3 K_6 \end{bmatrix}. \quad (5b)$$

Definition 1 [19] Consider the system (1a–1g) and cost function (2a–2d), if there exist a control law represented by $u^*(i,j)$ and a positive scalar represented by J^* such that for all admissible uncertainties, the closed-loop system (4) is asymptotically stable. Also the closed-loop value of the cost function (5) satisfies $J \le J^*$, then J^* is considered to be a guaranteed cost and $u^*(i,j)$ is exclaimed to be a guaranteed cost control law for the system (1a–1g).

The following important results are to be recalled on the stability of 2-D discrete uncertain system described by the General model with shift delays.

Lemma 1 [21] *The system (1a-1g) is globally asymptotically stable if and only if*

$$\det\{I_n - Z_1 Z_2 (A_3 + LF(i,j)M_{13}) - Z_2(A_2 + LF(i,j)M_{12}) - Z_1(A_1 + LF(i,j)M_{11} \\ - Z_1^{d+1} Z_2^{k+1}(A_{3dk} + LF(i,j)M_{13dk}) - Z_2^{k+1}(A_{2k} + LF(i,j)M_{12k}) - Z_1^{d+1}(A_{1d} + LF(i,j)M_{11d})\} \ne 0 . (6)$$

for all $(Z_1, Z_2, F(i,j)) \in \overline{U^2}$, where $\overline{U^2} = \{(Z_1, Z_2, F(i,j)) : |Z_1| \le 1, |Z_2| \le 1, \|F(i,j)\| \le 1\}$.

Remark 1

With $[\Delta A_1 \ \Delta A_{1d} \ \Delta A_2 \ \Delta A_{2k} \ \Delta A_3 \ \Delta A_{3dk}] = L\,F(i,j)[M_{11} \ M_{11d} \ M_{12} \ M_{12k} \ M_{13} \ M_{13dk}] = 0,$ is identified as global asymptotic stability condition [21] of the nominal 2-D discrete shift delayed system described by the General model.

Lemma 2 [21] The *closed-loop system (4) is globally asymptotically stable, provided there exist* $n \times n$ *positive definite symmetric matrices* $P, P_1, P_2, P_3, P_4, and\ P_5$ *such that*

$$\tau = [A_{\Delta 1} + B_{\Delta 1}K_1 \quad A_{\Delta 1d} + B_{\Delta 1}K_2 \quad A_{\Delta 2} + B_{\Delta 2}K_3 \quad A_{\Delta 2k} + B_{\Delta 2}K_4 \quad A_{\Delta 3} + B_{\Delta 3}K_5 \quad A_{\Delta 3dk} + B_{\Delta 3}K_6]^T \\ \times P\,[A_{\Delta 1} + B_{\Delta 1}K_1 \quad A_{\Delta 1d} + B_{\Delta 1}K_2 \quad A_{\Delta 2} + B_{\Delta 2}K_3 \quad A_{\Delta 2k} + B_{\Delta 2}K_4 \quad A_{\Delta 3} + B_{\Delta 3}K_5 \quad A_{\Delta 3dk} + B_{\Delta 3}K_6]$$

$$-\begin{bmatrix} P-P_1-P_2-P_3-P_4-P_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_5 \end{bmatrix} < 0 \quad \textit{for all } \|F(i,j)\| \le 1 . \qquad (7)$$

where

$A_{\Delta 1} = A_1 + \Delta A_1$, $A_{\Delta 1d} = A_{1d} + \Delta A_{1d}$, $A_{\Delta 2} = A_2 + \Delta A_2$,

$A_{\Delta 2k} = A_{2k} + \Delta A_{2k}$, $A_{\Delta 3} = A_3 + \Delta A_3$, $A_{\Delta 3dk} = A_{3dk} + \Delta A_{3dk}$,

$B_{\Delta 1} = B_1 + \Delta B_1$, $B_{\Delta 2} = B_2 + \Delta B_2$, $B_{\Delta 3} = B_3 + \Delta B_3$.

On the basis of the above mentioned lemma, the following definition is presented.

Definition 2 If there exists a $6n \times 6n$ positive definite symmetric matrices P, P_1, P_2, P_3, P_4 and P_5 such that it satisfies:

$$\tau + W_2 < 0 \quad \textit{for all} \quad \|F(i,j)\| \le 1, \qquad (8)$$

then control law for memory state feedback (3) is defined to be a quadratic guaranteed cost control associated with the cost matrix $0 < P = P^T \in R^{n\times n}$ for the system (4) and cost function (5).

The following lemma is needed in the proof of the main results.

Lemma 3 [17] *Let* $A \in R^{n\times n}$, $L \in R^{n\times p}$, $M \in R^{q\times n}$ *and* $Q_t = Q_t^T \in R^{n\times n}$ *be given matrices, and there exists a positive definite matrix* P *which fulfills*

$$[A + LFM]^T P [A + LFM] + Q_t < 0 . \qquad (9)$$

for all F *satisfying* $F^T F \le I$, *if and only if there exists a scalar* $\varepsilon > 0$ *such that*

$$\begin{bmatrix} -P^{-1} + \varepsilon LL^T & A \\ A^T & \varepsilon^{-1} M^T M + Q_t \end{bmatrix} < 0 . \qquad (10)$$

Main Results

In this section a relation with the upper bound of the closed loop system (4) and quadratic stability is established using the notion of cost matrix.

Lemma 4 *If there exist* $n \times n$ *positive definite symmetric matrices* $P, P_1, P_2, P_3, P_4, P_5$ *for the system (4) with the initial condition (1f-1g) and cost function (5) such that (8) holds then (i) system (4) is quadratic ally stable and (ii) the closed-loop cost function satisfies the bound*

$$J \le \{(r_1 - 1)\lambda\max(M^T(P - P_1 - P_2 - P_3 - P_4 - P_5)M) + (r_1 - 1)(d_1 + 1)\lambda\max(M^T P_1 M)$$

$$+(r_2-1)\lambda\max(M^T P_2 M)+(r_2-1)(k_1+1)\lambda\max(M^T P_3 M)+(r_1+r_2-1)\lambda\max(M^T P_4 M)$$
$$+[(1+d_2)(1+k_2)-(r_1-1)(r_2-1)]\lambda\max(M^T P_5 M)\}. \quad (11)$$

for all admissible uncertainties.

***Proof**:* **Lemma 2** and **definition 2** defines proof *of* (i).

In order to prove (ii), consider a quadratic 2-D Lyapunov function represented by

$$v(x(i,j)) = x^T(i,j)Px(i,j). \quad (12)$$

Let $\Delta v(x(i,j))$ be defined as

$$\Delta v(x(i,j)) = x^T(i+1,j+1)Px(i+1,j+1) - x^T(i,j+1)(P-P_1-P_2-P_3-P_4-P_5)x(i,j+1)$$
$$-x^T(i-d_1,j+1)P_1x(i-d_1,j+1) - x^T(i+1,j)P_2x(i+1,j) - x^T(i+1,j-k_1)P_3x(i+1,j-k_1)$$
$$-x^T(i,j)P_4x(i,j) - x^T(i-d_2,j-k_2)P_5x(i-d_2,j-k_2). \quad (13)$$

Along with the trajectory of the closed-loop system (4), we obtain

$$\Delta v(i,j) = \xi_{ij}^T \tau \xi_{ij}. \quad (14)$$

where ξ_{ij} and τ are defined in (2b) and (7), respectively. Since P is a cost matrix, it is followed from **Definition (2)** that

$$\xi_{ij}^T(\tau + W_2)\xi_{ij} < 0. \quad (15)$$

From (14) and (15), we have

$$\xi_{ij}^T W_2 \xi_{ij} < -\Delta v(x(i,j)). \quad (16)$$

Summing both sides of the equation (13) over $i,j = 0$ to ∞ yields

$$J = -\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\Delta v\, x(i,j) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}[x^T(i,j+1)(P-P_1-P_2-P_3-P_4-P_5)x(i,j+1)$$
$$-x^T(i+1,j+1)(P-P_1-P_2-P_3-P_4-P_5)x(i+1,j+1)$$
$$+x^T(i-d_1,j+1)P_1x(i-d_1,j+1) - x^T(i+1,j+1)P_1x(i+1,j+1)$$
$$+x^T(i+1,j)P_2x(i+1,j) - x^T(i+1,j+1)P_2x(i+1,j+1)$$
$$+x^T(i+1,j-k_1)P_3x(i+1,j-k_1) - x^T(i+1,j+1)P_3x(i+1,j+1)$$
$$+x^T(i,j)P_4x(i,j) - x^T(i+1,j+1)P_4x(i+1,j+1)$$
$$+x^T(i-d_2,j-k_2)P_5x(i-d_2,j-k_2) - x^T(i+1,j+1)P_5x(i+1,j+1)]. \quad (17)$$

Applying the initial conditions (1f)

$$J \le \sum_{j=1}^{r_1-1}\left[x^T(0,j)(P-P_1-P_2-P_3-P_4-P_5)x(0,j)\right] + \sum_{j=1}^{r_1-1}\sum_{i=-d_1}^{0}\left[x^T(i,j)P_1x(i,j)\right]$$

$$+\sum_{i=1}^{r_2-1}\left[x^T(i,0)P_2x(i,0)\right]+\sum_{i=1}^{r_2-1}\sum_{j=-k_1}^{0}\left[x^T(i,j)P_3x(i,j)\right]+\sum_{i=0}^{r_2-1}\sum_{j=0}^{r_1-1}\left[x^T(i,j)P_4x(i,j)\right]$$
$$-\sum_{i=1}^{r_2-1}\sum_{j=1}^{r_1-1}\left[x^T(i,j)P_4x(i,j)\right]+\sum_{i=-d_2}^{0}\sum_{j=-k_2}^{0}\left[x^T(i,j)P_5x(i,j)\right]-\sum_{i=1}^{r_2-1}\sum_{j=1}^{r_1-1}\left[x^T(i,j)P_5x(i,j)\right]. \quad (18)$$

Using the conditions given in (1g)

$$J\le\{(r_1-1)\lambda\max(M^T(P-P_1-P_2-P_3-P_4-P_5)M)+(r_1-1)(d_1+1)\lambda\max(M^TP_1M)$$
$$+(r_2-1)\lambda\max(M^TP_2M)+(r_2-1)(k_1+1)\lambda\max(M^TP_3M)+(r_1+r_2-1)\lambda\max(M^TP_4M)$$
$$+[(1_1+d_2)(1+k_2)-(r_1-1)(r_2-1)]\lambda max(M^TP_5M)\}. \quad (19)$$

This completes the proof of **lemma 4**.

Theorem 1 *Consider a system (1a-1g),having initial conditions and cost function defined (2a-2d), then there exists a memory state feedback control law of the form shown in (3) that resolves the addressed robust guaranteed cost control problem if there exist a positive scalar* $\in$, $m\times n$ *matrices* U_1,U_2,U_3,U_4,U_5,U_6 *and* $n\times n$ *positive definite symmetric matrices* V,Y_1,Y_2,Y_3,Y_4 *and* Y_5 *such that the following LMI becomes feasible:*

$$\begin{bmatrix}\Pi_1 & \Pi_2\\ * & \Pi_3\end{bmatrix}<0, \quad (20)$$

where

$$\Pi_1=\begin{bmatrix}(-V+\varepsilon LL^T) & (A_1V+B_1U_1) & (A_{1d}V+B_1U_2) & (A_2V+B_2U_3) & (A_{2k}V+B_2U_4) & (A_3V+B_3U_5) & (A_{3dk}V+B_3U_6)\\ (A_1V+B_1U_1)^T & -V+Y_1+Y_2+Y_3+Y_4+Y_5 & 0 & 0 & 0 & 0 & 0\\ (A_{1d}V+B_1U_2)^T & 0 & -Y_1 & 0 & 0 & 0 & 0\\ (A_2V+B_2U_3)^T & 0 & 0 & -Y_2 & 0 & 0 & 0\\ (A_{2k}V+B_2U_4)^T & 0 & 0 & 0 & -Y_3 & 0 & 0\\ (A_3V+B_3U_5)^T & 0 & 0 & 0 & 0 & -Y_4 & 0\\ (A_{3dk}V+B_3U_6)^T & 0 & 0 & 0 & 0 & 0 & -Y_5\end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ V\overline{M}_{11}{}^T & U_1{}^T R_1^{1/2} & VS_1^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ V\overline{M}_{11d}{}^T & U_2{}^T R_1^{1/2} & \mathbf{0} & VS_2^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ V\overline{M}_{12}{}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & U_3{}^T R_2^{1/2} & VS_3^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ V\overline{M}_{12k}{}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & U_4{}^T R_2^{1/2} & \mathbf{0} & VS_4^{1/2} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ V\overline{M}_{13}{}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & U_5{}^T R_3^{1/2} & VS_5^{1/2} & \mathbf{0} \\ V\overline{M}_{13dk}{}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & U_6{}^T R_3^{1/2} & \mathbf{0} & VS_6^{1/2} \end{bmatrix},$$

$$\Pi_3 = diag\{-\varepsilon I, I, I, I, I, I, I, I, I, I\},$$

$$\overline{M}_{11} = M_{11} + M_{21}K_1,\ \overline{M}_{11d} = M_{11d} + M_{21}K_2,\ \overline{M}_{12} = M_{12} + M_{22}K_3,$$
$$\overline{M}_{12k} = M_{12k} + M_{22}K_4,\ \overline{M}_{13} = M_{13} + M_{23}K_5,\ \overline{M}_{13dk} = M_{13dk} + M_{23}K_6,$$
$$V^{-1} = P,\ P^{-1} = V,\ VP_1V = Y_1,\ VP_2V = Y_2,\ VP_3V = Y_3,\ VP_4V = Y_4,\ VP_5V = Y_5. \tag{21}$$

Here the stabilizing control law matrices is given by

$$K_1 = U_1V^{-1},\ K_2 = U_2V^{-1},\ K_3 = U_3V^{-1},\ K_4 = U_4V^{-1},\ K_5 = U_5V^{-1},\ K_6 = U_6V^{-1}. \tag{22}$$

Moreover, closed-loop cost function satisfies the bound

$$J \le [(r_1-1)+(r_1-1)(d_1+1)+(r_2-1)+(r_2-1)(k_1+1)$$
$$+(r_1+r_2-1)+(1+d_2)(1+k_2)-(r_1-1)(r_2-1)]\lambda max(M^T PM). \tag{23}$$

Proof: Using (1b-1g), (5a-5b), **Lemma 3**, (8), (10) can be rearranged as

$$\begin{bmatrix} -P^{-1}+\varepsilon LL^T & (A_1+B_1K_1) & (A_{1d}+B_1K_2) & (A_2+B_2K_3) & (A_{2k}+B_2K_4) & (A_3+B_3K_5) & (A_{3dk}+B_3K_6) \\ (A_1+B_1K_1)^T & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{11}+K_1^TR_1K_1+S_1-P_1 & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{11d}+K_1^TR_1K_2 & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{12} & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{12k} & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{13} & \varepsilon^{-1}\overline{M}_{11}{}^T\overline{M}_{13dk} \\ (A_{1d}+B_1K_2)^T & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{11}+K_2^TR_1K_1 & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{11d}+K_2^TR_1K_2+S_2-P_2 & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{12} & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{12k} & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{13} & \varepsilon^{-1}\overline{M}_{11d}{}^T\overline{M}_{13dk} \\ (A_2+B_2K_3)^T & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{11} & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{11d} & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{12}+K_3^TR_2K_3+S_3-P_3 & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{12k}+K_3^TR_2K_4 & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{13} & \varepsilon^{-1}\overline{M}_{12}{}^T\overline{M}_{13dk} \\ (A_{2k}+B_2K_4)^T & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{11} & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{11d} & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{12}+K_4^TR_2K_3 & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{12k}+K_4^TR_2K_4+S_4-P_4 & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{13} & \varepsilon^{-1}\overline{M}_{12k}{}^T\overline{M}_{13dk} \\ (A_3+B_3K_5)^T & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{11} & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{11d} & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{12} & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{12k} & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{13}+K_5^TR_3K_5+S_5-P_5 & \varepsilon^{-1}\overline{M}_{13}{}^T\overline{M}_{13dk}+K_5^TR_3K_6 \\ (A_{3dk}+B_3K_6)^T & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{11} & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{11d} & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{12} & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{12k} & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{13}+K_6^TR_3K_5 & \varepsilon^{-1}\overline{M}_{13dk}{}^T\overline{M}_{13dk}+K_6^TR_3K_6+S_6-P_6 \end{bmatrix}. \tag{24}$$

Pre-multiplying and post-multiplying (24) by the $diag\{I, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}\}$ and applying Schur, complement we can obtain (20)

Further, to satisfy (20) it necessarily requires that

$$V - (Y_1 + Y_2 + Y_3 + Y_4 + Y_5) \geq 0 . \quad (25)$$

Equation (25) can be rewritten as

$$V^{-1} \geq V^{-1}(Y_1 + Y_2 + Y_3 + Y_4 + Y_5)V^{-1} . \quad (26)$$

From (25) and (26), we obtain

$$\lambda max(M^T V^{-1} M) \geq \lambda max(M^T V^{-1}(Y_1 + Y_2 + Y_3 + Y_4 + Y_5)V^{-1} M) . \quad (27)$$

Now this also means that

$$\lambda max(M^T PM) \geq \lambda max(M^T (P_1 + P_2 + P_3 + P_4 + P_5)M) . \quad (28)$$

which leads to

$$\lambda max(M^T PM) \geq \lambda max(M^T P_1 M),\ \lambda max(M^T PM) \geq \lambda max(M^T P_2 M),$$

$$\lambda max(M^T PM) \geq \lambda max(M^T P_3 M), \lambda max(M^T PM) \geq \lambda max(M^T P_4 M),$$

$$\lambda max(M^T PM) \geq \lambda max(M^T P_5 M) . \quad (29)$$

Now, following the similar steps given in (25-29), the bound of cost function can be easily obtained from (11) as,

$$J \leq [(r_1 - 1) + (r_1 - 1)(d_1 + 1) + (r_2 - 1) + (r_2 - 1)(k_1 + 1)$$

$$+ (r_1 + r_2 - 1) + (1 + d_2)(1 + k_2) - (r_1 - 1)(r_2 - 1)]\lambda max(M^T PM) .$$

This completes the proof of **Theorem 1**.

Remark 2

It is to be pointed out that the matrix inequality (20) is linear in variables $\varepsilon, U_1, U_2, U_3, U_4, U_5, U_6, V, Y_1, Y_2, Y_3, Y_4$ and Y_5. Therefore, Mat lab LMI Toolbox [26, 27] can be used to find the feasible solution of (20) if it exists. Theorem 1 presents a method of designing the memory state feedback guaranteed cost controllers (if they exist) by (20) in terms of the feasible solution. It also helps in illustrating the way of selecting suitable controllers minimizing the guaranteed cost in (23).

Remark 3

It may be noted that, **Theorem1** can also be used to design guaranteed cost controllers via memory less state feedback for 2-D discrete shift-delayed systems described by the General Model simply by putting $K_2 = K_4 = K_6 = 0$.

Remark 4

The above theorem may also be reduced to provide the method for designing guaranteed cost controllers for various other 2-D discrete state space models, particularly when $(A_{1d}+\Delta A_{1d})=\mathbf{0}$, $(A_{2k}+\Delta A_{2k})=\mathbf{0}$, $(A_3+\Delta A_3)=\mathbf{0}$, $(A_{3dk}+\Delta A_{3dk})=\mathbf{0}$, $(B_3+\Delta B_3)=\mathbf{0}$ along with $K_2=K_4=K_6=\mathbf{0}$, **Theorem 1**coincides with the existing results given in [11, 12].

Further if $(A_3+\Delta A_3)=\mathbf{0}$, $(A_{3dk}+\Delta A_{3dk})=\mathbf{0}$, $(B_3+\Delta B_3)=\mathbf{0}$ together with $K_5=K_6=\mathbf{0}$, **Theorem 1**coincides with the existing results of [19]. Along with above condition if K_2 and K_4 are also made $\mathbf{0}$ **Theorem 1** coincides with the existing results in [17, 18].

Also if $(A_{1d}+\Delta A_{1d})=\mathbf{0}$, $(A_{2k}+\Delta A_{2k})=\mathbf{0}$, $(A_{3dk}+\Delta A_{3dk})=\mathbf{0}$, along with $K_2=K_4=K_6=\mathbf{0}$ **Theorem 1** coincides with the existing results of 2-D discrete system given in [13, 20].

Theorem 2 [11, 19] *In the system description along with the initial conditions (1a-1g) and cost function (2a-2d),if the following optimization problem:*

minimize λ

$$s.t.\begin{cases}(i).\ \ (20),\\ (ii).\ \begin{bmatrix}-\lambda I & M^T\\ M & -V\end{bmatrix}<0.\end{cases} \quad (30)$$

has a feasible solution $\lambda>0, \in>0, m\times n$ *matrices* $U_1, U_2, U_3, U_4, U_5, U_6$ *and* $n\times n$ *positive definite symmetric matrices* V, Y_1, Y_2, Y_3, Y_4 *and* Y_5 *,then the control law of structure represented by (3)is the optimal guaranteed cost control law which ensures the minimization of guaranteed cost given by (23).*

Proof: By **Theorem 1**, the stabilizing control law matrices (3) constructed in terms of any feasible solution $\varepsilon, V, Y_1, Y_2, Y_3, Y_4, Y_5, U_1, U_2, U_3, U_4, U_5$ and U_6 are the guaranteed cost controllers of system (1a-1g). To achieve the optimum value of the upper bound of guaranteed cost, the term $\lambda max\,(M^T V^{-1} M)$ in (27) is changed to $\lambda max\,(M^T V^{-1} M)<\lambda I$, which in turn implies the constraint (ii) of (30). This resulted in the minimization of the guaranteed cost in (23). This completes the proof of **Theorem 2**.

Remark 5 It may be noted that **Theorem 2** coincides with the existing results of [11-13, 19] for the design of optimal controllers, under the similar conditions given in **Remark 4**.

Illustrative example

The thermal process of a heat exchanger is expressed in the partial differential equation as

$$\frac{\partial T(x,t)}{\partial x} = -\frac{\partial T(x,t)}{\partial t} - a_0 T(x,t) - a_1 T(x - x_d, t) - a_2 T(x, t-\tau) + bu(x,t)\,. \qquad (31)$$

Denoting $x^T(i,j) = [T^T(i-1,j)\;\; T^T(i,j)]$; $\mathbf{int}\left(\frac{x_d}{\Delta x}\right) \to d$; $\mathbf{int}\left(\frac{\tau}{\Delta t}\right) \to (k+1)$ (where int(.) is the integer function), it is easy to verify that the equation (31) can be converted into General model delayed (1a) with parameter matrices where $T(x,t)$ is usually the temperature at x (space) $\in [0, x_f]$ and t (time) $\in [0,\infty], u(x,t)$ is a given force function, τ is the time delay, x_d is the space delay, a_0, a_1, a_2, b are real coefficients.

Proof: $\boldsymbol{T}(i,j) = \boldsymbol{T}(i\Delta x, j\Delta t)\,, \boldsymbol{u}(i,j) = \boldsymbol{u}(i\Delta x, j\Delta t)$

$$\frac{\partial \boldsymbol{T}(x,t)}{\partial x} \approx \frac{\boldsymbol{T}(i+1,j) - \boldsymbol{T}(i,j)}{\Delta x}\,, \frac{\partial \boldsymbol{T}(x,t)}{\partial t} \approx \frac{\boldsymbol{T}(i,j) - \boldsymbol{T}(i,j-1)}{\Delta t}\,. \qquad (32)$$

Denoting $x^T(i,j) = [\boldsymbol{T}^T(i-1,j)\;\; \boldsymbol{T}^T(i,j)]$, $\mathbf{int}\left(\frac{x_d}{\Delta x}\right) \to d, \mathbf{int}\left(\frac{\tau}{\Delta t}\right) \to (k+1)$

(whereint (.) is the integer function).

$$\boldsymbol{T}(i\Delta x + \Delta x, j\Delta t) = \boldsymbol{T}(i\Delta x, j\Delta t)\left[1 - \left(\frac{\Delta x}{\Delta t}\right) - a_0 \Delta x\right] + \left(\frac{\Delta x}{\Delta t}\right)\boldsymbol{T}(i\Delta x, j\Delta t - \Delta t) - a_1 \Delta x\, \boldsymbol{T}\left(i\Delta x - \mathbf{int}\left(\frac{x_d}{\Delta x}\right)\Delta x, j\Delta t\right)$$

$$- a_2 \Delta x\, \boldsymbol{T}\left(i\Delta x, j\Delta t - \mathbf{int}\left(\frac{\tau}{\Delta t}\right)\Delta t\right) + b\Delta x u(i\Delta x, j\Delta t)\,. \qquad (33)$$

The value of parametric matrices can be easily calculated as shown in (33) after a simple rearrangement.

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & \left(1 - \left(\frac{\Delta x}{\Delta t}\right) - a_0 \Delta x\right) \end{bmatrix},\; \boldsymbol{A}_{1d} = \begin{bmatrix} 0 & 0 \\ 0 & -a_1 \Delta x \end{bmatrix},\; \boldsymbol{A}_2 = \begin{bmatrix} 0 & 0 \\ \left(\frac{\Delta x}{\Delta t}\right) & 0 \end{bmatrix},$$

$$\boldsymbol{A}_{2k} = \begin{bmatrix} 0 & 0 \\ -a_2 \Delta x & 0 \end{bmatrix},\; \boldsymbol{A}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\; \boldsymbol{A}_{3dk} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\; \boldsymbol{B}_1 = \begin{bmatrix} 0 \\ b\Delta x \end{bmatrix},\; \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\; \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \qquad (34)$$

Let $\Delta t = 0.415,\;\; \Delta x = 0.361,\;\; a_0 = 3.18,\;\; a_1 = 0.147,\;\; a_2 = 0.0421,\;\; b = 1\,.$ (35)

Using the data values of (35), the parametric matrices values can be calculated, with

$$\boldsymbol{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -1.02 \end{bmatrix},\; \boldsymbol{A}_{1d} = \begin{bmatrix} 0 & 0 \\ 0 & -0.0533 \end{bmatrix},\; \boldsymbol{A}_2 = \begin{bmatrix} 0 & 0 \\ 0.87 & 0 \end{bmatrix},\; \boldsymbol{A}_{2k} = \begin{bmatrix} 0 & 0 \\ -0.0152 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\; \boldsymbol{A}_{3dk} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\; \boldsymbol{B}_1 = \begin{bmatrix} 0 \\ 0.361 \end{bmatrix},\; \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{B}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \qquad (36)$$

Further we assume

$$\boldsymbol{M}_{11} = [0.02 \quad -0.25], \boldsymbol{M}_{11d} = [0.02 \quad 0.05], \quad \boldsymbol{M}_{12} = [0 \quad 0.005], \boldsymbol{M}_{12k} = [0.01 \quad 0.03], \boldsymbol{M}_{13} = [0 \quad 0],$$
$$\boldsymbol{M}_{13dk} = [0 \quad 0], \boldsymbol{M}_{21} = [0.21], \boldsymbol{M}_{22} = [0.18], \quad \boldsymbol{M}_{23} = [0], \boldsymbol{S}_1 = diag\{0.018,\ 0.018\}, \boldsymbol{S}_2 = diag\{0.016,\ 0.016\},$$
$$\boldsymbol{S}_3 = diag\{0.028,\ 0.028\}, \boldsymbol{S}_4 = diag\{0.009,\ 0.009\}, \quad \boldsymbol{S}_5 = \boldsymbol{S}_6 = diag\{0,0\}, \boldsymbol{R}_1 = \boldsymbol{R}_2 = [0.026], \quad \boldsymbol{R}_3 = [0],$$
$$r_1 = 2,\ r_2 = 3,\ d_1 = 3,\ d_2 = 2,\ k_1 = 2,\ k_2 = 1, \boldsymbol{L} = \begin{bmatrix} 0.12 \\ 0.16 \end{bmatrix}, \quad \boldsymbol{M} = \begin{bmatrix} 0.21 \\ 0.15 \end{bmatrix}. \tag{37}$$

Using Lemma 1, it is easy to verify that the above system is unstable. The optimal guaranteed cost controllers along with the other optimized parameters are calculated by solving Theorem 2 using MATLAB LMI toolbox [26, 27] and is given by

$$\boldsymbol{V} = \begin{bmatrix} 0.2400 & -0.0018 \\ * & 0.2035 \end{bmatrix}, \boldsymbol{U}_1 = [-0.0050 \quad 0.5472], \boldsymbol{U}_2 = [-0.0005 \quad 0.0292], \boldsymbol{U}3 = [0.0482 \quad -0.0060],$$

$$\boldsymbol{U}_4 = [-0.0139 \quad -0.0338], \boldsymbol{U}_5 = [0 \quad 0], \boldsymbol{U}_6 = [0 \quad 0], \lambda = 0.2966,$$
$$\boldsymbol{K}_1 = [-0.0010 \quad 2.6882], \quad \boldsymbol{K}_2 = [-0.0010 \quad 0.1436], \quad \boldsymbol{K}_3 = [0.2008 \quad -0.0278],$$
$$\boldsymbol{K}_4 = [-0.0591 \quad -0.1667], \boldsymbol{K}_5 = [0 \quad 0], \quad \boldsymbol{K}_6 = [0 \quad 0]. \tag{38}$$

Using the optimized value of the λ, the least upper bound of the corresponding closed-loop cost function is

$$J < 6.2286 . \tag{39}$$

Conclusions

In this paper, a method to design optimal guaranteed cost controllers using memory state feedback for uncertain 2-D discrete shift delayed systems described by the General model is presented. For the existence of controllers, first LMI based sufficient condition has been established and then a convex optimization problem with LMI constraints is formulated to achieve the optimal controllers. Finally, an example of thermal process with delays in time and space is provided to illustrate the effectiveness of the proposed approach.

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