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Correlation between Safe Bearing Capacity and Angle of Internal Friction of Pilani Soil

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Abstract

Safe bearing capacity of soils is used in foundation design. Shallow foundations in the form of continuous strips are widely used. Standard equations are available to estimate the safe bearing capacity of soil supporting strip shallow foundations. It appears from these equations that there is a strong dependence of bearing capacity on angle of internal friction. An attempt has been made in the present study to get simplified correlation between safe bearing capacity of soil and angle of internal friction using the principle of least squares. Correlation has been developed by analyzing 20 different soil samples and verified for 19 other soil samples. Analytical equations in the form of linear as well as quadratic polynomial have been developed. Correlation coefficient between actual and estimated value of safe bearing capacity for linear and quadratic polynomial was found to be 0.869319 & 0.942516.

Keywords

Safe bearing capacity, Shallow foundations, Soil, Angle of internal friction, Correlation, Principle of least squares, Analytical equations.

Introduction

Foundation is a substructure element. This element transmits the structural load to the soil such that there is no shear failure of soil. Furthermore, settlement of foundation is also under control. Foundations are divided as shallow and deep. Shallow foundations are located below ground surface at depths less than or at most equal to the width of the foundation.

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Shallow Foundations are subdivided based on the geometric shape of their base. Following equations are used to determine ultimate bearing capacity of shallow foundation if foundation base is a continuous strip" [1], loading is vertical, eccentricity effect is not present and depth factor is ignored:

$q_d = cN_c + qN_q + 0.5B\gamma N_\gamma$ (Under general shear failure) (1)

$q_d = 0.67cN'_c + qN'_q + 0.5B\gamma N'_\gamma$ (Under local shear failure) (2)

Where,
q_d = Ultimate bearing capacity in kg/cm^2
N_c, N_q & N_γ = Bearing capacity factors under general shear failure
N'_c, N'_q & N'_γ = Bearing capacity factors under local shear failure
c = Cohesion of soil in kg/cm^2
q = Effective surcharge at the base level of the footing in kg/cm^2 ($=\gamma D_f$)
γ = Unit weight of soil in kg/cm^3
D_f = Depth of embedment of footing from ground level in cm
B = Width of strip footing in cm

These equations assume water table location at considerable depth from foundation. If water table is in the vicinity of foundation, a correction is employed to equations (1) and (2). For most of the soils, either general or local shear failure is applicable. For some soils, interpolation between general and local shear failure is applicable. Punching shear failure is rarely encountered.

Bearing capacity factors depend on the angle of internal friction of soil" [2]. Consequently, ultimate bearing capacity as well as safe bearing capacity should also depend on angle of internal friction for a given foundation width and depth in a particular region where there is not much variation in the density of soil.

An attempt has been made in the present study to get a relation between safe bearing capacity and angle of internal friction. This relation has been developed based on the correlation observed between safe bearing capacity and angle of internal friction of soil samples collected within 100km from Birla Institute of Technology & Science; Pilani campus. Method of least squares has been used to obtain the correlation. Angle of internal friction has been taken as the independent variable and safe bearing capacity as the dependent variable. Equations for linear as well as quadratic curve of best fit have been developed based on the principle of least squares. These equations can be used to estimate safe bearing capacity of soil by knowing its angle of internal friction value.

Development of Least Square Equations

Least square equations in the present study have been developed for linear as well as quadratic interrelation between safe bearing capacity and angle of internal friction. Twenty soil samples have been taken for this purpose. Foundations for most of the structures in the region of study is located 2meters below ground surface. Consequently, all the 20 soil samples have been collected from a depth of 2meters below ground surface by constructing test pit of 2meter depth below ground surface at each location. Strip type of shallow foundation is applicable for most of the structures in the region of study. Consequently, safe bearing capacity for shallow foundation has been obtained under strip footing conditions at a depth of 2meters below ground surface keeping the footing width equal to 2meters. Ultimate bearing capacity has been calculated first using design equations available in specifications" [1], incorporating the effect of water table. The loading is vertical with no eccentricity effect. Depth factor has been ignored. The value of ultimate bearing capacity thus obtained has been divided by factor of safety of 3 to get safe bearing capacity. These design equations for general and local shear failure conditions are given as equation (1) and equation (2) respectively. Values of safe bearing capacity (SBC) for these 20 soil samples along with their angle of internal friction (ϕ) value (obtained experimentally) are given in Table 1. They have also been plotted in Figure 1.

No.	Safe bearing capacity (SBC) (T/m^2)	Angle of internal friction (ϕ) (deg.)
1	5.20	23.62
2	6.49	25.66
3	7.00	26.73
4	7.37	27.27
5	7.72	27.63
6	7.93	27.55
7	8.02	27.65
8	8.25	27.21
9	8.43	27.43
10	8.57	28.15
11	8.80	29.03
12	8.88	28.41
13	9.23	29.24
14	9.34	29.49
15	9.67	29.13
16	9.76	29.60
17	9.92	29.11
18	10.72	30.89
19	10.86	30.32
20	12.68	39.42

Table 1: Safe bearing capacity and Angle of internal friction value for 20 soil samples

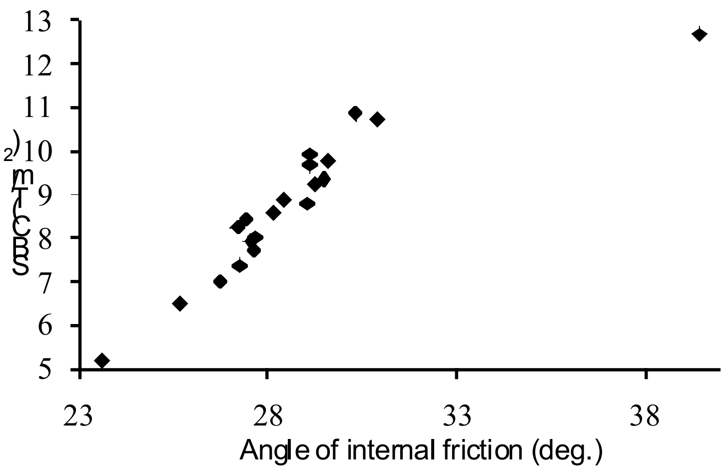

Figure 1: Interrelation between SBC and Angle of internal friction

From Figure 1, it is evident that there is a definite correlation between safe bearing capacity (T/m^2) and angle of internal friction for the 20 soil samples listed in Table 1. As a first approximation in the present study, a linear relation has been tried. Consider:

$SBC = a\phi + b$ (3)

Following least square technique, the quantity 'S' has to be minimized, where:

$S = \sum[SBC_i - (a\phi_i + b)]^2$ (4)

In order to minimize 'S', its derivative with respect to 'a' and 'b' should be zero. Performing these on equation (4), following are obtained:

$nb + a\sum\phi_i = \sum SBC_i$ (5)

$b\sum\phi_i + a\sum\phi^2_i = \sum\phi_i\, SBC_i$ (6)

For 20 soil samples of Table 1, in all the summations of equations (5) and (6), 'i' will range from 1 to 20. For 20 soil samples of Table 1, $\sum\phi_i = 573.54$, $\sum\phi^2_i = 16620.69$, $\sum SBC_i = 174.84$ & $\sum\phi_i\, SBC_i = 5100.621$. Furthermore, 'n' = 20 for 20 soil samples of Table 1. Substituting these values in equations (5) and (6), following equations are obtained:

$20b + 573.54a = 174.84$ (7)

$573.54b + 16620.69a = 5100.621$ (8)

Equations (7) and (8) are a system of two linear equations with two unknowns. Solving these equations for 'a' and 'b', we get: a = 0.5005 & b = -5.6118. These solutions have been obtained using Matlab" [3]. Since equations (7) and (8) are independent, values of 'a' and 'b' are unique. Substituting these values of 'a' and 'b' in equation (3):

$SBC = 0.5005\phi - 5.6118$ (9)

A quadratic relation between 'SBC' and 'ϕ' has been also developed in the present study taking 'ϕ' as independent and 'SBC' as dependent variable. Consider:

$SBC = a\phi^2 + b\phi + c$ (10)

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The unknown constants ′a′, ′b′ and ′c′ have been determined using least square technique for which the quantity 'S1' should be minimized, where:

$$S1 = \sum[SBC_i - (a\phi^2_i + b\phi_i + c)]^2 \quad (11)$$

To minimize ′S1′ of equation (11), its derivative with respect to ′a′, ′b′ and ′c′ should be equated to zero. Performing these on equation (11) and on simplifying, we get:

$$a\sum\phi^4_i + b\sum\phi^3_i + c\sum\phi^2_i = \sum\phi^2_i\, SBC_i \quad (12a)$$

$$a\sum\phi^3_i + b\sum\phi^2_i + c\sum\phi_i = \sum\phi_i\, SBC_i \quad (12b)$$

$$a\sum\phi^2_i + b\sum\phi_i + cn = \sum SBC_i \quad (12c)$$

For 20 soil samples of Table 1, in all the summations of equations (12a), (12b) and (12c), ′i′ will range from 1 to 20. For 20 soil samples of Table 1, $\sum\phi_i = 573.54$, $\sum\phi^2_i = 16620.69$, $\sum\phi^3_i = 487650.7$, $\sum\phi^4_i = 14518974$, $\sum SBC_i = 174.84$, $\sum\phi_i\, SBC_i = 5100.621$, $\sum\phi^2_i\, SBC_i = 150619.4$ & ′n′ = 20. Substituting these in equations (12a), (12b) & (12c):

$$14518974a + 487650.7b + 16620.69c = 150619.4 \quad (13a)$$

$$487650.7a + 16620.69b + 573.54c = 5100.621 \quad (13b)$$

$$16620.69a + 573.54b + 20c = 174.84 \quad (13c)$$

Equations (13a) to (13c) are three linear independent equations in three unknowns which can be solved using Matlab to get the values for ′a′ = -0.0329, ′b′ = 2.5956 & ′c′ = -38.3124. Substituting these in equation (10):

$$SBC = -0.0329\phi^2 + 2.5956\phi - 38.3124 \quad (14)$$

Equations (9) and (14) can be used to estimate safe bearing capacity (SBC) of soils whose angle of internal friction value is known and collected from a depth of 2meters below ground level from the region within 100km from Birla Institute of Technology & Science, Pilani campus. The estimated safe bearing capacity is for 2meters wide strip footing under vertical loading with no eccentricity and with depth factor ignored.

Verification of Developed Equations

Applicability of equations (9) and (14) have been verified using 19 soil samples. These 19 soil samples were also collected from a depth of 2meters below ground surface and from locations within 100km of Birla Institute of Technology & Science, Pilani campus. Table 2 lists angle of internal friction value (obtained experimentally) of these soil samples and their SBC value obtained as per specifications incorporating the effect of water table (for 2meter wide strip footing at 2meter depth below ground surface). The loading is vertical with no eccentricity. Depth factor has been ignored. Furthermore, Table 2 also lists their SBC values using equations (9) and (14). SBC (linear), refers to SBC obtained using equation (9) and SBC(quadratic) refers to SBC obtained using equation (14) in Table 2.

No.	ϕ (deg.)	SBC(T/m^2)	SBC (linear)(T/m^2)	SBC(quadratic)(T/m^2)
1	24.88	6.56	6.84064	5.900554
2	25.09	6.48	6.945745	6.100388
3	26.32	7.28	7.56136	7.212567
4	26.83	7.67	7.816615	7.644519
5	26.95	8.03	7.876675	7.743668
6	27.18	7.77	7.99179	7.931054
7	27.45	8.41	8.126925	8.146588
8	27.69	8.46	8.247045	8.334146
9	27.81	7.98	8.307105	8.426504
10	27.96	8.70	8.38218	8.540619
11	28.48	9.47	8.64244	8.924756
12	28.74	9.26	8.77257	9.110152
13	29.08	9.16	8.94274	9.345881
14	29.09	9.82	8.947745	9.3527
15	29.12	9.68	8.96276	9.373114
16	29.18	8.80	8.99279	9.413766
17	30.16	10.78	9.48328	10.04421
18	30.46	10.59	9.63343	10.22457
19	30.89	11.12	9.848645	10.47276

Table 2: ϕ, SBC, SBC (linear) & SBC(quadratic) for 19 soil samples

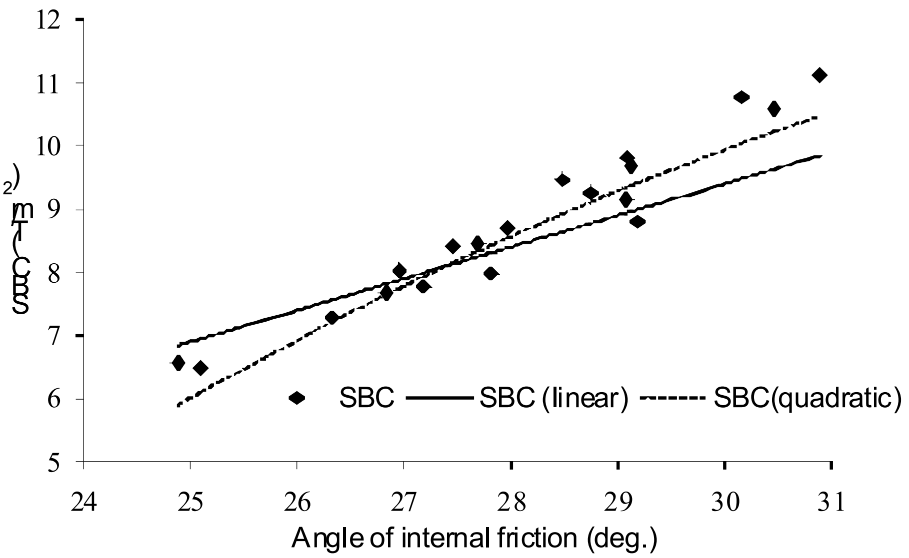

Figure 2: Variation of SBC, SBC (linear) & SBC (quadratic) with Angle of internal friction

Variations of SBC, SBC (linear) and SBC(quadratic) with angle of internal friction of soil has also been plotted in Figure 2. From Figure 2 it is clear that there exists a correlation between

SBC value computed as per specifications as well as SBC values obtained from equations (9) and (14). Consequently, equations (9) and (14) can be used to estimate safe bearing capacity for a soil of known angle of internal friction value, collected from a depth of 2meter below ground surface within 100km from Birla Institute of Technology & Science, Pilani campus.

Furthermore, correlation coefficient between SBC v/s SBC (linear) as well as SBC v/s SBC(quadratic) for 19 soil samples of Table 2 also have been determined. Correlation coefficient (r) is determined as follows" [4]:

$$r = [1 - \frac{\sigma^2_{SBC,\phi}}{\sigma^2_{SBC}}]^{1/2} \quad (15)$$

Where,

$$\sigma_{SBC} = [\frac{\Sigma(SBC_i - SBC_m)^2}{(n-1)}]^{1/2} \quad (16a)$$

and,

$$\sigma_{SBC,\phi} = [\frac{\Sigma(SBC_i - SBC_{ic})^2}{(n-m-1)}]^{1/2} \quad (16b)$$

In all the summations in equations (16a) and (16b), 'i' will range from 1 to 19 for 19 soil samples of Table 2. 'SBC_m' is the arithmatic mean of 19 'SBC' values of Table 2 and its value is 8.737895. 'SBC_i' is the SBC value and 'SBC_{ic}' is corresponding SBC (linear) [for SBC v/s SBC (linear) correlation] or SBC(quadratic) [for SBC v/s SBC(quadratic) correlation] value for a given angle of internal friction value of Table 2 with 'i' ranging from 1 to 19. $\sum(SBC_i - SBC_m)^2$ for 19 soil samples of Table 2 has value of 31.62972. $\sum(SBC_i - SBC_{ic})^2$ for 19 soil samples of Table 2 has value of 7.297452 for SBC v/s SBC (linear) correlation and is equal to 3.139476 for SBC v/s SBC(quadratic) correlation. 'n' represents number of soil samples and has value of 19 for 19 soil samples of Table 2. 'm' in equation (16b) represents degree of polynomial and has value of 1 for linear polynomial and has value of 2 for quadratic polynomial. Consequently, 'm' = 1 for SBC v/s SBC(linear) correlation & = 2 for SBC v/s SBC(quadratic) correlation. Substituting these values in equation (15), correlation coefficient comes out to be 0.869319 for SBC v/s SBC (linear) correlation and 0.942516 for SBC v/s SBC(quadratic) correlation. Consequently, out of equations (9) and (14), equation (14) will give better estimate of safe bearing capacity.

Conclusions

In the present study simple equations have been developed to estimate safe bearing capacity. Angle of internal friction can be determined experimentally and one can directly use equations (9) and (14) to estimate safe bearing capacity. Since these equations have been developed based on analysis of 20 soil samples within 100km of Birla Institute of Technology & Science, Pilani campus, where soil density variation is not much, they will provide satisfactory estimate for soil samples from the same region. Furthermore, since SBC v/s SBC(quadratic) has higher correlation coefficient than SBC v/s SBC (linear), equation (14) will provide better estimate of safe bearing capacity than equation (9). For soil samples of other region, there could be variation in density and one can perform similar type of analysis as presented in this study to develop simplified equations to estimate safe bearing capacity as a function of angle of internal friction for soil samples of that region. Since only angle of internal friction information is needed, to estimate safe bearing capacity, the estimation work becomes quite simplified. Hence, it can be concluded that the equations developed in present study are of considerable significance. Estimated value of safe bearing capacity using equations developed in present study are for 2meters wide strip footing with depth of embedment 2meters. Loading condition is vertical with no eccentricity effect. Depth factor has been ignored.

Acknowledgement

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