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Analysis of a Linear Consecutive k-out-of-N System with Common Cause Shock Failure

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Abstract

The present study deals with the analysis of a linear consecutive k-out-N system with the common cause shock failure. The system consists of n units arranged in a linear way out which when k consecutive units fail, the system will fail, otherwise system works properly. Also it is assumed, the life time and the repair time of the units are distributed exponentially. The system may be fail due to the failure of individual units or due to the common cause shock failure. Common cause failure is a phenomena in which all components of the working system are failed with a common reasons such as fire, power outage etc.. In this system, the transition probabilities of transient as well as steady states are determined which are used to reliability indices with such as availability and mean time to failure on the basis of Markov model. A numerical illustration is also carried out to validate the model.

Keywords

Linear consecutive k-out-of-N system, Availability, Common cause shock failure, Mean time to failure.

Introduction

Reliability of the system becomes more prominent in various areas such as communication system, manufacturing system, computer, electronic system and transportation system etc. The k-out-of-n system is the most important form of repairable system according to reliability theory. The consecutive k- out- of- n: F repairable system, which is found in linear form and circular form. The suitable example of this type system is oil pipeline system and telecommunication system etc. The example of circular consecutive k-out-of-n-system can be seen in water supply system. In the recent past, a few research works in this field have been published by Zhang et al. [1] and Cheng and Zhang [2].

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In a circular consecutive k-out-of-N: F, a closed recurring water supply system with n water pumps in a thermo-electric plant has been presented by Yam et al. [3]. Bolend et al. [4] described a linear (circular) consecutive k-out-of-n: F system. The life time distribution of consecutive k-out-of-n: F system has been explained by Eryılmaz [5]. Lu and Lewis [6] determined a configuration for k-out-of-n partially redundant systems. The reliability properties of consecutive k-out-of-n systems of arbitrarily dependent components are studied by Eryılmaz [7]. Salehi et al. [8] did reliability analysis of consecutive *k*-out-of-*n* systems with non-identical components lifetimes. Eryılmaz and Navarro [9] obtained some mixture representations for both linear and circular consecutive systems. Yuan and Cui [10] considered the systems which had one repairman without vacation or infinite repairmen without vacations.

The remaining paper is organized as follows; section 2 deals with the model description and the assumptions used to develop the model. In section 3 the reliability and availability analysis of the model have been done. A numerical example is given in section 4 to validate the model. In the last conclusion is drawn in section 5.

Model Description

Consider a linear consecutive 2-out-of-3: F repairable system with common cause shock failure. In this system, the system works if 2 or more units work properly but after the failure of 2 or more units the system stop working as shown in fig. 1.The following assumptions are made to formulate the model.

1. The failure of each unit of system of the system is independent and inertial.
2. When the k consecutive units fail, the system will fail.
3. The two possible states of the system and the units are working and failed states.
4. The units of the life time and the repair time are exponentially distributed with parameter λ and   respectively.
5. Initially, all the system units are in working state.
6. The system may be fail due to the common cause shock failure which is exponentially distributed with the parameter 'γ'.

Some notation which are used to develop the model are as follows-

λ; the failure rate of the units

; the repair rate of the units

γ; common cause shock failure

$_b$; the faster repair rate of the total system

P (t); probability of the system at time t

$\widetilde{P}_n(s)$; Laplace transform of $P_n(t)$ = [n=0, 1, 2, 3]

P_n; steady state probability at time t of n^{th} state [n=0, 1, 2, 3]

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The Analysis

In this section the reliability indices such as availability and mean time to failure are obtained for both steady and transient states of the system.

The Reliability

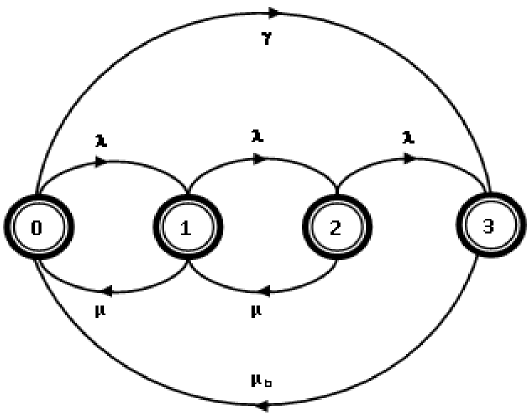

Figure 1: State transition diagram for reliability analysis

The transient state equations governing the model are constructed as follows:

$$\frac{dP_0(t)}{dt} = -(\lambda+\gamma)P_0(t) + \mu P_1(t) + \mu P_3(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -(\lambda+\mu)P_1(t) + \mu P_2(t) + \lambda P_0(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -(\lambda+\mu)P_2(t) + \lambda P_1(t) \quad (3)$$

$$\frac{dP_3(t)}{dt} = -\mu_b P_3(t) + \lambda P_2(t) + \gamma P_0(t) \quad (4)$$

Let initially conditions are

$$P_0(t)=1, P_1(t)=0, P_2(t)=0, P_3(t)=0, P_4(t)=0$$

After taking the Laplace transform of equations

$$s\tilde{P}_0(s) - 1 = -(\lambda+\gamma)\tilde{P}_0(s) + \mu\tilde{P}_1(s) + \mu\tilde{P}_3(s) \quad (5)$$

$$s\tilde{P}_1(s) = -(\lambda+\mu)\tilde{P}_1(s) + \mu\tilde{P}_2(s) + \lambda\tilde{P}_0(s) \quad (6)$$

$$s\tilde{P}_2(s) = -(\lambda+\mu)\tilde{P}_2(s) + \lambda\tilde{P}_1(s) \quad (7)$$

$$s\tilde{P}_3(s) = \mu_b\tilde{P}_3(s) - \lambda\tilde{P}_2(s) + \gamma\tilde{P}_0(s) \quad (8)$$

Now solving the equations, (5)-(8), we get, The reliability function of the system is given by

$\tilde{R}(s) = \tilde{P}_0(s) + \tilde{P}_1(s) + \tilde{P}_2(s)$

$$= \frac{\lambda^2(s+\mu_b) + \left[(s+\lambda+\mu)^2 - \lambda\mu\right] (s+\mu_b) + \lambda(s+\lambda+\mu)(s+\mu_b)}{(s+\lambda+\gamma)(s+\lambda+\mu)^2(s+\mu_b) - \lambda\mu(s+\lambda+\gamma)(s+\mu_b) - \lambda^3\mu_b - \mu_b\gamma(s+\lambda+\mu) + \lambda\mu\gamma\mu_b - \lambda\mu(s+\lambda+\mu)(s+\mu_b)} \tag{9}$$

The mean time to system failure (MTTF) is obtained by

$$\text{MTTF} = \lim_{s\to 0}\tilde{R}(s) = \frac{\lambda^2\mu_b + \left[(\lambda+\mu)^2 - \lambda\mu\right]\mu_b + \lambda(\lambda+\mu)\mu_b}{\mu_b(\lambda+\gamma)(\lambda+\mu)^2 - \lambda\mu\mu_b(\lambda+\gamma) - \mu_b\lambda^3 - \gamma\mu_b(\lambda+\mu)^2 + \lambda\mu\gamma\mu_b - \lambda\mu\mu_b(\lambda+\mu)} \tag{10}$$

The Availabilty

The equations of the steady state of a system are given by

$$-(\lambda+\gamma)P_0 + \mu P_1 + \mu_b P_3 = 0 \tag{11}$$

$$-(\lambda+\mu)P_1 + \mu P_2 + \lambda P_0 = 0 \tag{12}$$

$$-(\lambda+\mu)P_2 + \lambda P_1 = 0 \tag{13}$$

$$-\mu_b P_3 + \lambda P_2 + \gamma P_0 = 0 \tag{14}$$

Normalizing conditions are,

$$P_0 + P_1 + P_2 + P_3 = 1 \tag{15}$$

On solving equations (11)-(15), the availability of a system is obtained by

$$A(\infty) = P_0 + P_1 + P_2 = \frac{\mu_b(\lambda+\mu)^2 - \lambda\mu\mu_b + \mu\mu_b(\lambda+\mu) + \lambda\mu\mu_b}{(\lambda+\mu)^2\mu_b - \lambda\mu_b + \mu\mu_b(\lambda+\mu) + \lambda\mu\mu_b + \lambda^2\mu - \gamma(\lambda+\mu)^2 + \lambda\mu\gamma} \tag{16}$$

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Numerical Illustration

In this section, we find the performance measures of the model with the help of the analytical result those obtained from the transient and steady state equations. We have considered an example of a telecommunication system in which there are three relay channels are connected in a linear manner. For the smooth running of the system the functioning of two channels is highly recommended. The system will be fail when two consecutive relay channels fail. For computational purpose, the coding of the program has been done in software 'MATLAB'. We fix μ_b =2 as default parameter and evaluate the numerical results by varying λ, μ and γ. The numerical results are summarized in figures 2-5 to explore the effect of failure rate λ and repair rate μ on reliability indices such as availability and MTTF. In figures 2-4, the decreasing pattern of availability and MTTF is observed for increasing values of λ for different values of γ. It is noticed that the availability and MTTF increase with the increasing values of μ for different values of γ as shown in fig. 3 and 5. Based on numerical results, we conclude that the availability and MTTF decrease (increase) as λ (μ) increases.

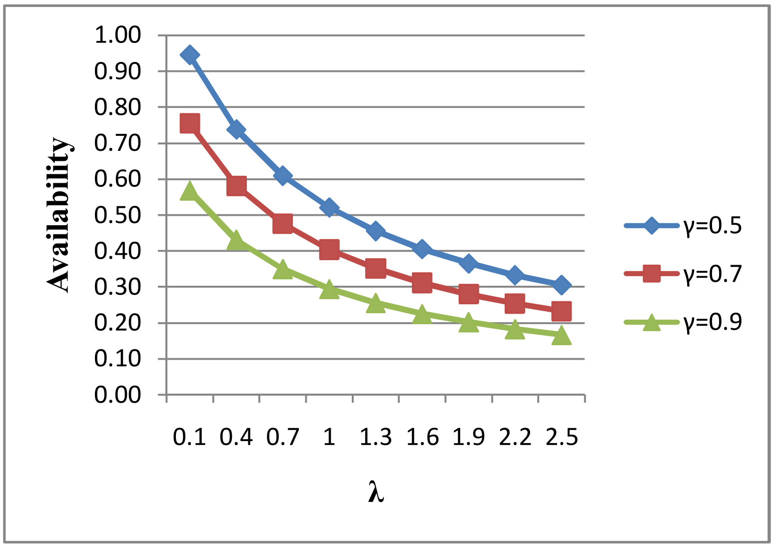

Figure 2: Availability vs λ for different values of γ

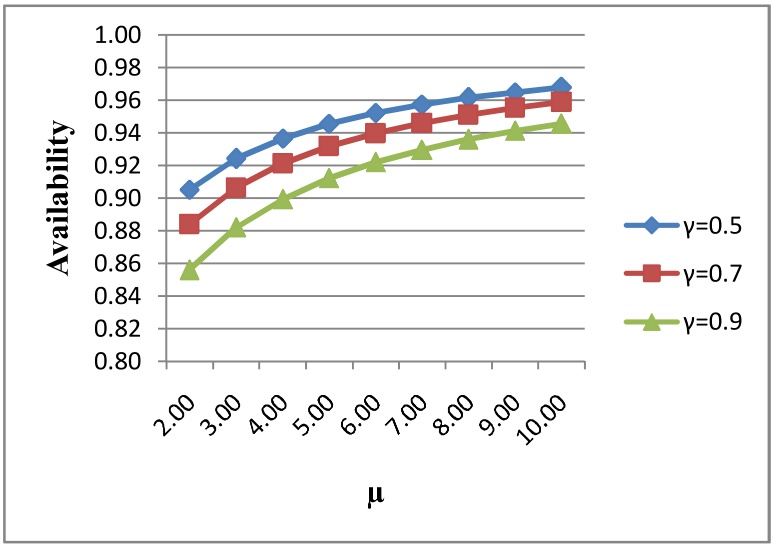

Figure 3: Availability vs μ for different values of γ

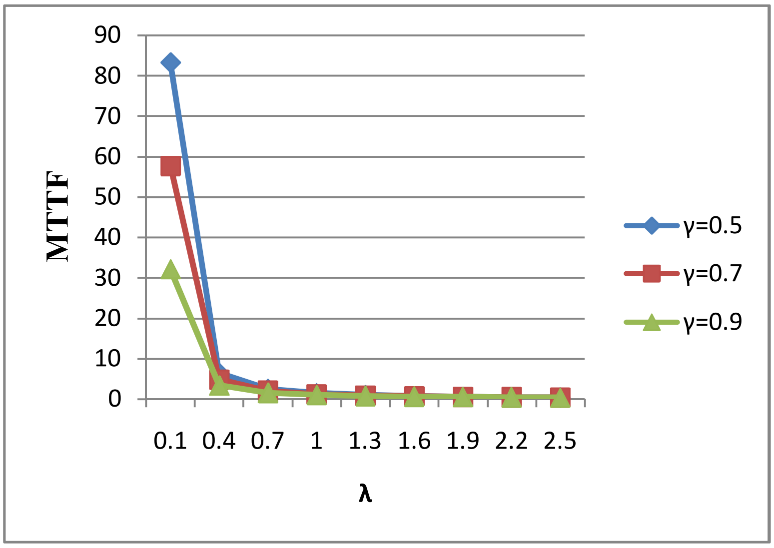

Figure 4: MTTF vs λ for different values of γ

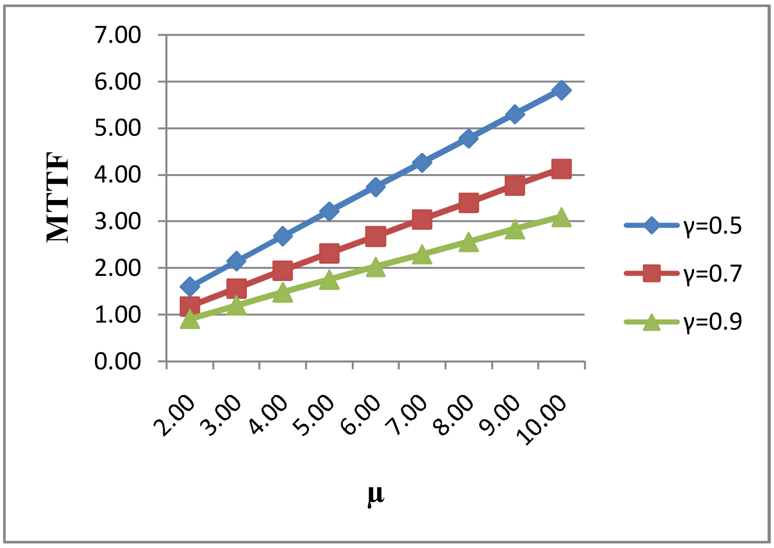

Figure 5: MTTF vs μ for different values of γ

Conclusion

In this paper, consider a repairable system with common cause shock failure, imperfect converge, reboots and recovery. This paper determined the transient equations of the reliability and steady equations of the availability. The reliability indices provide a powerful tool for reliability analysis of a linear consecutive k-out-of-N: F repairable systems. The reliability indices obtained may be helpful to the system designers to evaluate more useful and precise information about system characteristics which can be further exploit for the development and improvement of the system concerned.

References

[1] Y. L. Zhang, , M. J. Zuo and , R. C. M. Yam, “Reliability analysis for a circular consecutive-2- out-of-n:F repairable system with priority in repair” Reliab. Engg. Sys. Safety (2000), vol. 68, pp. 113–120.

[2] K. Cheng and Y. L. Zhang, “Analysis for a consecutive- k-out-of-n:F repairable system with priority in repair” Inter. J. Sys. Sci. (2001), vol. 32, pp. 591–598.

[3] R. C. M. Yam, M. J. Zuo and Y. L. Zhang, “A method for evaluation of reliability indices for repairable circular consecutive k-out-of-N: F systems” Reliab. Engg. Sys. Safety (2003), Vol. 79, no. 1, pp 1-9.

[4] P. J. Boland and F. J. Samaniego, “Stochastic ordering results for consecutive k-out-of-N: F systems” IEEE Trans. Reliab. (2004), vol. 53, no. 1, pp. 7-10.

[5] S. Eryılmaz, “On the life time distribution of consecutive k-out-of-n:F system” IEEE Trans. Reliab. (2007), vol. 56, pp. 35–39.

[6] L. Lu and G. Lewis, “Configuration determination for K-out-of-n partially redundant systems” Reliab. Engg. Sys. Safety (2008), nol. 93, no. 11, pp. 1594-1604.

[7] S. Eryılmaz, “Reliability properties of consecutive k-out-of-n systems of arbitrarily dependent components” Reliab. Engg. Sys. Safety (2009), vol. 94, pp. 350–356.

e-ISSN: 2321-1814, ISBN (Print): 978-1-943730-43-8

[8] E.T. Salehi, M. Asadi and S. Eryılmaz, "Reliability analysis of consecutive k-out-of-n systems with non-identical components lifetimes" J. Stat. Plan. Infer. (2011), vol. 141, no. 8, pp. 2920-2932.

[9] S. Eryılmaz and J. Navarro., "Failure rates of consecutive k-out-of-n systems" J. Korean Stat. Soc. (2012), vol. 41, no. 1, pp. 1-11.

[10] L. Yuan and Z-D. Cui, "Reliability analysis for the consecutive-k-out-of-n: F system with repairmen taking multiple vacations" Appl. Math. Model. (2013), vol. 37, pp.4685–4697.