

Channel Estimation using Iterative Process for OFDMA based Wireless System

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Abstract

In this paper the channel estimation and data detection for OFDMA based systems over time varying frequency selective channels was described using a modified superimposed training (SIT) sequence method with iterative process. This method can be significantly improved the accuracy by exploiting time and frequency correlations between the channel frequency response coefficients (coherence time and coherence bandwidth). In SIT based channel estimation scheme, training sequence is arithmetically added to the data sequence with a desired power allocation ratio at the transmitter. And at the receiver the channel estimation is done by using the same SIT sequence, further the mean square error (MSE) is minimized with iterative process from detected data symbols. This method improves the system performance in terms of MSE and bit error rate (BER) for higher order modulation and high mobility cases.

Keywords

LTE, OFDMA, SIT, coherence time, coherence bandwidth, MMSE.

Introduction

The wireless mobile and personal communication systems are expected to support a variety of high speed multimedia services, such as high speed internet access, high quality video transmission and so on. To meet the demand for these high data rate services in broadband wireless systems, various systems and technologies have been proposed, such as evolved 3G (termed Long Term Evolution (LTE)), and 4G mobile communication systems. The basic concept of LTE comes from orthogonal frequency division multiple access (OFDMA) even though the OFDMA system is far from being a trivial task. The channel estimation and synchronization are the supreme issues in OFDMA based systems. This is particularly true when the delay spread is more because in this scenario the coherent bandwidth becomes smaller and the interpolation becomes more difficult.

The Channel estimation in OFDMA and single carrier frequency division multiple access (SC-FDMA) is typically done using pilots placed intermittently along with data in the time frequency grid [1]-[4]. At low mobility these methods give good performance and also

reducing bandwidth efficiency of the system. If the mobility and data rate increases, the pilot spacing needs to be reduced to track the channel variations both in time and frequency. The problem is further aggravated when multi-input multi-output (MIMO) systems are incorporated both in the uplink and the down link. Hence there is a strong need for bandwidth efficient channel estimation in such systems with acceptable performance.

Improved channel estimate performance can be obtained via 2-D minimum mean square error (MMSE) channel estimation by taking into account the correlations along time and frequency that exist among the neighboring channel frequency response (CFR) coefficients [1],[2]. However, in most practical wireless or cellular multi-carriers systems, 2-D MMSE channel estimator based on the true underlying channel correlation sequence is hard to realize because of at least one of the following three reasons: First, the media access control (MAC) layer scheduler often changes the assigned block of sub-carriers to each user, thereby complicating the task of channel correlation function estimation. Second, the estimation of the correlation function itself incurs additional computational complexity. Third, an inaccurate estimate of the correlation function may lead to an uncontrolled degradation in terms of channel estimation mean square error (MSE) with no bound on the worst case performance.

Superimposed training (SIT) based channel estimation has been proposed recently for OFDMA based wireless communications, for its potential bandwidth efficiency [5], [6]. In this method the training symbols are superimposed on data symbols (arithmetically added to the data symbols) with desired power allocation ratio at the transmitter. By doing this, the data symbols can be transmitted over all time-frequency slots, hence saving bandwidth compared to conventional channel estimation methods. The channel is estimated using the SIT sequence. The estimator is generalized to extend the same to coherently integrate over several OFDM symbols experiencing the same fading coefficients of the channel. The performance of the channel estimate of the proposed scheme is analyzed in terms of the MMSE and the bit error rate (BER) of the OFDMA system.

Section II discusses the proposed SIT based channel estimation method. The performance of this method in terms of the minimum mean square error of the channel estimator is also discussed in same section. Section III gives the performance comparison with respect to existing methods and simulation results respectively and finally concluded in section IV.

Proposed Channel Estimation Method using Superimposed Training Sequence

In SIT based method, the sum powers of training and data symbols are constrained by a fixed transmit power budget. Consider a scenario where the data is detected by using estimated channel information at the receiver. Good data detection performance requires highly accurate channel estimate which favors more transmit power assigned to training symbols but, more power allocated to training symbols results in less leaving for data symbols, inversely causing degradation in data detection performance. Hence, a power allocation tradeoff between training and data symbols becomes crucial. The performance

of the system depends on the mutual interference of data symbols and training symbols, and also the correlation between the data symbols and SIT symbols, which deteriorates the channel estimation accuracy. This scheme is explained in the following sections.

Transmitter Model

Figure 1 illustrates the block diagram of OFDMA transmitter. In OFDMA transmitter, the data stream of each user is converted into parallel blocks of size N and each block is modulated by using an N-point inverse Fast Fourier transform (IFFT) and followed by cyclic prefix (CP) insertion to avoid interference between adjacent blocks

Generally, symbols before modulation are referred as frequency domain symbols, and symbols after modulation (passing IFFT) are called time-domain symbols. The output of IFFT block is given by

$$(1) \quad s(n) = \frac{1}{N} \sum_{k=0}^{N-1} S(k) e^{j2\pi nk/N} \quad n = 0, 1, 2, \dots, N-1$$

where S(k), k = 0, 1, 2, ..., N-1 represents the block of N complex data symbols chosen from an appropriate signal constellation such as quadrature amplitude modulation (QAM) or phase shift keying (PSK) and s(n) represents the OFDM symbol having N data symbols after IFFT block.

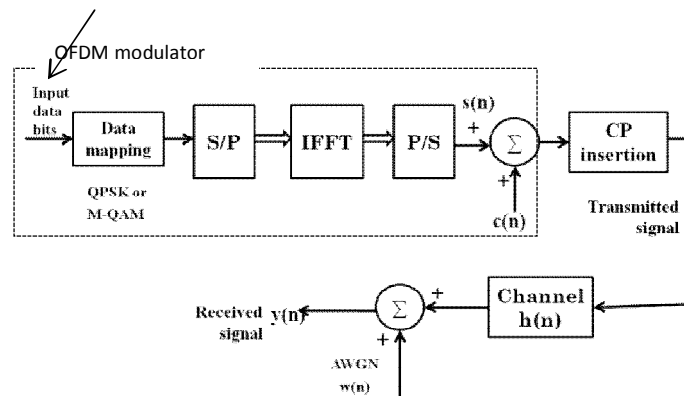


Figure1: Block diagram for OFDMA transmitter based on SIT (where S/P = serial to parallel converter and P/S = parallel to serial converter)

The superimposed training based OFDM transmitter model is illustrated as shown in Fig. 1. A training sequence c_k is algebraically added to the output of IFFT block at a prescribed data to training sequence power ratio $\alpha = \sigma_s^2 / \sigma_c^2$.

Where σ_s^2 and σ_c^2 are the total average power of the data and training sequence respectively.

Then, $x(n) = s(n) + c(n)$

After inserting cyclic prefix (CP) between the adjacent IFFT blocks, the OFDM symbols are transmitted over a time-varying frequency selective fading channel.

Channel Model

High-data rates in wireless communications give rise to inter-symbol interference (ISI) due to multipath fading. Such an ISI channel is called frequency-selective. On the other hand, the received signal is affected by Doppler shift due to mobility and hence time-variation. The combination of this time variation with ISI gives rise to so-called doubly selective channel (time varying frequency selective). The complex baseband impulse response of a time varying frequency selective fading channel $h(t, \tau)$ is given by

$$h(t, \tau) = \sum_{i=0}^{L-1} h_i(t) \delta(\tau - \tau_i) \quad (2)$$

Where, $h_i(t)$ is the time varying amplitude of the i^{th} propagation path, τ_i is the delay of the i^{th} path and L is the number of propagation paths (channel length).

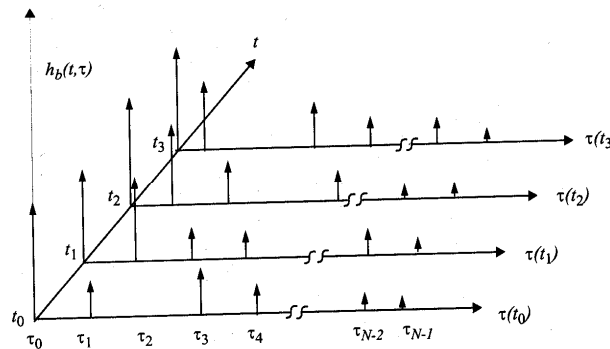


Figure 2: an example of the time varying discrete time impulse response model for multi-path radio channel [Ref.-9]

The amplitude of each path is assumed to be a Rayleigh process and the power delay profile of the channel is taken as exponentially decaying. The channel impulse response is modeled as a zero mean complex Gaussian distribution with uncorrelated paths. Each path fade coefficients are independently varying according to the Jake's power spectrum is given by

$$S(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left[\frac{f - f_m}{f_m} \right]^2}} \quad (3)$$

The time correlation function of the channel coefficients is $E[h_i(t_1)h_i^*(t_2)] = \sigma_i^2 J_0(2\pi f_d(t_2 - t_1))$ where σ_i^2 is the variance of the i th propagation path, $J_0(\cdot)$ is the 0th order of the Bessel function of the first kind and f_d is the Doppler frequency in Hz.

Receiver Model

The SIT based receiver model for OFDM system is shown in Fig.3. The baseband received signal at the receiver after CP removal is expressed as

$$y_k = x_k * h_k + w_k \quad (4)$$

Where, $x_k = s_k + c_k$, here $s_k = [s_0 \ s_1 \ s_2 \ \dots \ s_{N-1}]$ and $c_k = [c_0 \ c_1 \ c_2 \ \dots \ c_{N-1}]$ are data and training sequence vectors respectively, $h_k = [h_0 \ h_1 \ \dots \ h_{L-1}]$ is channel impulse response and w_k is AWGN associated with k th transmitted signal

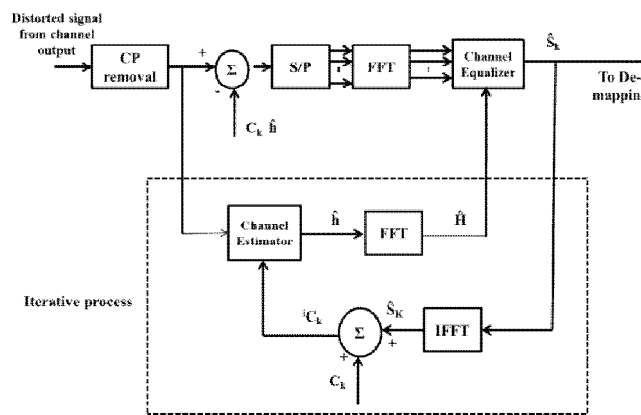


Figure 3: A schematic block diagram for OFDM receiver based on SIT

The equivalent baseband received vector after removal of CP can be expressed as:

$$(5) \quad y_k = X_k h_k + w_k$$

Where $X_k = S_k + C_k$, S_k and C_k are the Toeplitz matrices with dimension $N \times L$ of the data and training sequences respectively,

$$S_k = \begin{bmatrix} s_{k0} & 0 & 0 & \dots & 0 \\ s_{k1} & s_{k0} & 0 & \dots & 0 \\ \cdot & s_{k1} & 0 & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ s_{k(N-1)} & s_{k(N-2)} & \cdot & \dots & \cdot \end{bmatrix}_{N \times L}$$

h_k is channel vector of $L \times 1$ and w_k AWGN vector of $N \times 1$. Here k represents the OFDMA symbol number and L is the channel length, depends on the RMS delay spread.

The block fading nature of the channel encountered in practice in wireless communication systems allows us to assume that the channel is the same for several OFDM symbols, depending on the coherence time (T_c) of the channel. The definition of the coherence time implies that two signals arriving with a time separation is greater than T_c are affected differently by the channel. The coherence time is calculated by the following expression:

$$T_c = \frac{0.423}{f_d} \quad \text{where } f_d \text{ is the maximum Doppler shift}$$

The number of OFDMA symbols which are experienced nearly by the same channel

$$N_c = \frac{T_c}{T_s}$$

coefficients is

Where, T_s is one OFDMA symbol duration.

For formulation and performance analysis, the following assumptions are taken.

- Assume that perfect frequency synchronization and a zero DC offset at the baseband receiver.

- The data symbols are uncorrelated with each other and has zero mean (i.e. $E(S_k) = 0$), hence the second moment of the data symbols is the variance of the symbols

$$E(S_k S_k^H) = \sigma_s^2 I$$

- The data symbols and the training sequence are uncorrelated with each other

$$E(S_k^H C_k) = E(C_k^H S_k) = 0.$$

- Assume additive white Gaussian with variance $E(W_k W_k^H) = \sigma_w^2 I$, and mean zero $E(W_k) = 0$.

- The channel is wide sense stationary (WSS) and the relation given as

$$E(h_k h_k^H) = \text{diag}(E|h_0(n)|^2, E|h_1(n)|^2, \dots, E|h_{L-1}(n)|^2)$$

And the delay profile normalized to have unit area. Here $\text{diag}(\cdot)$ stands for the diagonal matrix and $(\cdot)^H$ stands for Hermitian matrix.

Channel Estimation using LS Method

The baseband received vector at receiver for k th OFDMA symbol is given by

$$y_k = S_k h_k + C_k h_k + w_k \quad (6)$$

The channel is estimated by LS method is given by

$$(7) \quad \hat{h}_k = [C_k^H C_k]^{-1} C_k^H y_k$$

Where \hat{h}_k : $L \times 1$ estimated channel coefficient vector. C_k is a matrix with dimension $N \times L$. The channel is constant for the duration of N_c received vectors (OFDMA symbols) hence; the improved estimation is obtained by averaging the estimated channel over N_c OFDMA symbols

$$\hat{h} = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \hat{h}_k \quad (8)$$

$$\hat{h} = \frac{1}{N_c} \left(\sum_{k=0}^{N_c-1} (C_k^H C_k)^{-1} C_k^H y_k \right) \quad (9)$$

In the process the estimator can be made to exploit the delay spread of the channel. An N point discrete Fourier transform (DFT) interpolation on the channel impulse response is performed to obtain the estimate of the frequency coefficients of the channel at all the sub-carriers with good performance. In this way the coherence time and coherence bandwidth of the channel are exploited to enhance the accuracy of the estimator.

The data symbols are recovered from the received symbols by equalization. Here, we use simple one tap equalizer. The data symbols are estimated by the following expression

$$\hat{S}_k = \frac{Z_k}{\hat{H}} \quad (10)$$

Where $Z_k = FFT\{y_k - C_k \hat{h}\}$ and \hat{H} is the FFT of estimated channel vector \hat{h} .

Iterative Process

Actually the estimated channel vector is expressed as follows

$$\hat{h}_k = h_k + (C_k^H C_k)^{-1} C_k^H (S_k h_k + w_k) \quad (11)$$

Where w_k is additive white Gaussian noise (AWGN) and which has very low correlation with other signals, thus we can ignore this term. Therefore the above expression can be written as

$$\hat{h}_k = h_k + (C_k^H C_k)^{-1} C_k^H S_k h_k = h_k + h_{err} \quad (12)$$

Where h_{err} is the error component (i.e., interference component on channel estimation due to data and training sequence), which depends on the cross correlation between data and training sequence. If the correlation is zero (i.e., data and training sequences are independent to each other), then the $h_{err} = 0$. But practically there is some correlation between data and training sequence. So, we need to reduce this interference for getting better channel estimate accuracy. The iterative method was proposed for improving the channel estimate accuracy.

The estimated symbol \hat{S}_k may be used to further improve the channel estimation accuracy in an iterative fashion [5]. This is because the interference of data symbols in the channel estimation is reduced in each iteration step. For iteration the estimated channel vector is

$${}^i \hat{h}_k = \left({}^i C_k^H {}^i C_k \right)^{-1} {}^i C_k^H y_k \quad (13)$$

The average estimation of the channel over N_c OFDMA symbol is

$$\hat{h} = \frac{1}{N_c} \left(\sum_{k=0}^{N_c-1} \left({}^i C_k^H {}^i C_k \right)^{-1} {}^i C_k^H y_k \right) \quad (14)$$

Where ${}^i C_k = {}^{i-1} \hat{S}_k + C_k$, ${}^0 C_k = C_k$ and ${}^i \hat{S}_k$ is the Toeplitz matrix of the time domain detected data symbol vector $IFFT\{\hat{S}_k\}$ in i^{th} iteration. Here, ${}^0 S_k = 0$

In this work, the another iterative method is proposed with a small modification in the above method, in which the interference on channel estimation due to data symbol is reduced by substituting the replica of detected data symbols in equation (12), and it can be written as

$$\hat{h}_k = \left(I + \left(C_k^H C_k \right)^{-1} C_k^H S_k \right) h_k \quad (15)$$

Now, find out the estimated actual channel impulse response by using the following expression

$$h_k = \left(I + \left(C_k^H C_k \right)^{-1} C_k^H S_k \right)^{-1} \hat{h}_k \quad (16)$$

By doing this, the channel estimation accuracy can be increased, but still there is some channel estimation error. Further the interference on channel estimation can be reduced by the iterative process explained above. This method improves the performance but more computational, time and circuit complexity compared to previous one.

Performance analysis: Mean square estimation error

The channel estimation error is expressed as $e = h - \hat{h}$ and the mean square estimation error is given by

$$(17) \quad \sigma_e^2 = \text{tr}\{E(ee^H)\}$$

where $\text{tr}\{\}$ stands for the trace of the matrix.

$$e = h_k - \left(I + \left(C_k^H C_k \right)^{-1} C_k^H S_k \right)^{-1} \left(C_k^H C_k \right)^{-1} C_k^H y_k$$
 by using the assumptions is given by above, the mean square error can be written as

$$\sigma_e^2 = (\sigma_w^2) \text{tr} \left\{ \left(\sum_{k=0}^{N_c-1} C_k^H C_k \right)^{-H} \right\} \quad (18)$$

Now minimize the mean square error under a power constraint of the training sequence in

(18) the inverse of the trace of the positive definite matrix $R_{cc} = \sum_{k=0}^{N_c-1} \left(C_k^H C_k \right)$ is to be minimized therefore the above expression can be expressed as

$$\sigma_e^2 = \sigma_w^2 \sum_{i=0}^{L-1} C_{ii} \quad (19)$$

without modification the mean square error [5]

$$\sigma_e^2 = (\sigma_s^2 + \sigma_w^2) \sum_{i=0}^{L-1} C_{ii}$$
 (20) Where C_{ii} is the i th diagonal element of the inverse of the R_{cc} given by,

$$C_{ii} = \frac{\text{Cof}(C_{ii})}{|R_{cc}|} \quad (21)$$

Where $\text{Cof}(C_{ii})$ is the cofactor of the i th diagonal element $C_{ii} = NN_c \sigma_c^2$ for all $[0, L-1]$. We use the following property of positive definite matrices,

$$0 < |R_{cc}| \leq \frac{C_{ii}}{\text{Cof}(C_{ii})} \quad (22)$$

$$R_{cc} = \sum_{k=0}^{N_c-1} \left(C_k^H C_k \right) = NN_c \sigma_c^2 I \quad (23)$$

where I is the identity matrix.

From equation (19) and (22) the MMSE can be written as

$$\sigma_{e \min}^2 = \frac{\sigma_w^2 L}{NN_c \sigma_c^2} \quad (24)$$

From (20) and (22) the MMSE can be written as

$$\sigma_{e \min}^2 = \frac{(\sigma_s^2 + \sigma_w^2)L}{NN_C \sigma_C^2} \quad (24)$$

Hence the training sequence that minimizes the mean square under the power constraints satisfies the following property

According to equation (24), the MMSE decreases with increasing N_c , which is limited by the coherent time and also MMSE increases with the channel order L , which is depend on the delay spread of the channel. Also we see that the MMSE is proportional to noise power and inversely proportional to training sequence power. So, the channel estimation accuracy is increased by increasing the training sequence power. If the power assigned to training sequence is increased then the power assigned to data will be low, hence the performance of the system will be reduced. Therefore there is tradeoff between the power allocation ratio and the system performance.

Optimal training sequence

The training sequence that is added to the data is the samples of a digitized linear frequency modulated (LFM) signal. The LFM sequence occupies all the sub-bands of the OFDM system with equal powered components. This training sequence is a periodic repetition is defined as follows

$$c(n) = c(n + N) = e^{j \frac{2\pi n}{N} \left(1 + \frac{n}{2}\right)} \quad (25)$$

Where N is the length of the training sequence and $n = 0, 1, 2, \dots, N-1$. This sequence is optimal training sequence in terms of minimizing the MSE and the BER of the OFDM system [10]. The sequence has a constant magnitude in the time domain. Such sequences are known to be optimal in terms of the PAPR of the aggregate of the data and training signals as discussed in [11].

Simulation Results

The following simulation parameters are considered while doing the simulations

TABLE 1: Simulation parameters

Parameter	Value
Carrier frequency	3GHz
Channel bandwidth	20MHz
Spacing between subcarriers	15KHz
Number of occupied subcarriers	1200
IFFT/FFT size	2048
Modulation format	QPSK

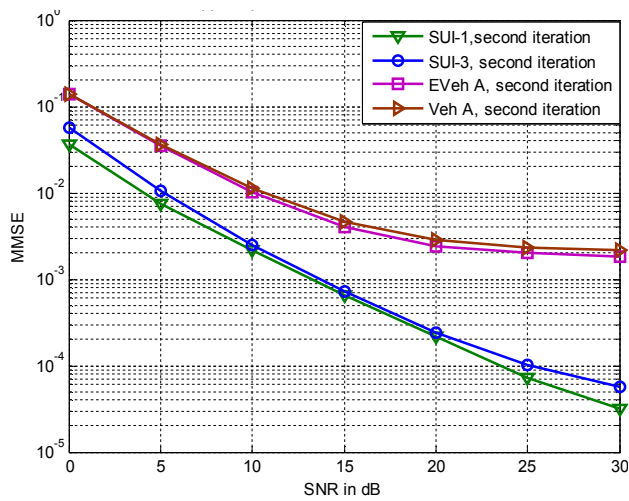


Figure 3: Signal to noise ratio (SNR) versus minimum mean square error (MMSE) for SIT based method for different Stanford University interim (SUI) channel models with Doppler spread 333.33Hz

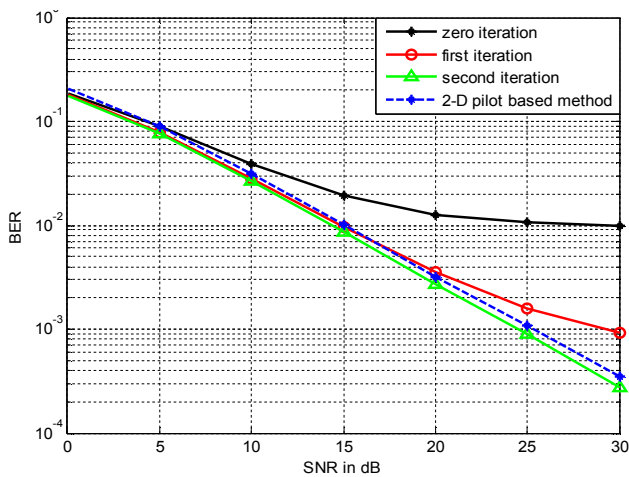


Figure 4: Signal to noise ratio (SNR) versus bit error rate (BER) comparison, for SUI-3 with Doppler spread 333.33Hz

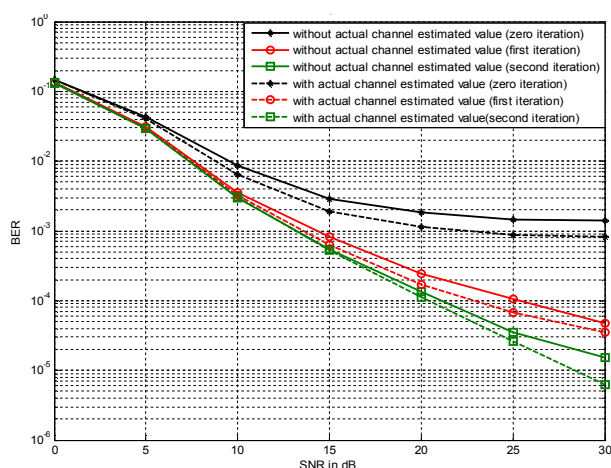


Figure 5: Signal to noise ratio (SNR) versus bit error rate (BER) for SIT based method at power ratio=5, SUI-1 channel model

The first plot shows the variation of MMSE in accordance with SNR for different types of SUI channel models. The second and third plots show the performance comparison of BER versus signal to noise ratio (SNR) between SIT based method without modification and 2-D pilot based method, and the modified SIT based method and SIT based method without modification. From this it is clearly shown that the proposed method gives the better channel estimation compared to others. If the acceptable BER of the receiver is 0.0002, then the minimum required SNR is equal to 26 dB in case of modified SIT based method and 29 dB in case of SIT based method without modification from the simulated results. Thus, the power of the transmitted signal can be saved up to 3 dB by using the proposed method compared to existing methods.

Conclusion

In this work the channel impulse response is estimated by using superimposed training based channel estimation scheme with modified iterative process over time-variant frequency selective fading channel for LTE air interface. This scheme is bandwidth efficient and spectral efficient compared to 2-D pilot based method. And also it is better in terms of power constraints when compared to other methods.

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