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Finsler Space Admitting Torse-Forming Vector Field

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Abstract

Torse-forming infinitesimal transformation $\bar{x}^i = x^i + v^i(x)dt, \quad v^i_{lj} = v^i \mu j + \alpha \delta^i_j$ and torse-forming curvature inheritance in a Finsler space have been studied by Mishra and Mishra[2], Mishra and Lodhi[3] respectively. They established some basic and useful results under Torse-forming infinistesimal transformation.

The object of present paper is to study the Torse-forming infinitesimal transformation for cartan's curvature. We have established the condition under which Cartan's curvature generates non-flat space under aforesaid Torse-forming infinitesimal transformation. Some useful results have been investigated in the paper.

Keywords

Finceler space, Cartan'sCurvatue, Torse-forming Transformation.

Introduction

Let us consider an n-dimensional an affinely connected finsler space Fn [4] with a symmetric connection parameter $\Gamma^{*i}_{jk}(x,\dot{x})$.

The covariant derivative of $T^i_j(x,\dot{x})$ with respect to xk in the sense of Cartan's is given by

(1.1) $$T^i_{j|k} = \partial_k T^i_j - \dot{\partial}_m T^i_j \; G^m_k + T^m_j \Gamma^{*i}_{mk} - T^i_m \Gamma^{*m}_{jk}$$

(1.2) $$T^i_{l|j} = \partial_j T^i_l + T^r_l G^i_{rj} - T^i_r G^r_{lj}$$

Where |j denote the h-covariant differentiation

(1.3) $$T^i_{l|j} = \dot{\partial}_j T^i_l + T^r_l V^i_{rj} - T^i_r V^r_{lj}$$

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Cartan's connection coefficient $\Gamma^{*i}_{jk}(x,\dot{x})$ satisfy the following relations.

$$(1.4)\ \ (a)\ \ \dot{\partial}_h \Gamma^{*i}_{jk}\, \dot{x}^h = 0 \ \text{and}\ (b)\ \dot{\partial}_h \Gamma^{*i}_{jk} = \dot{\partial}_j \Gamma^{*i}_{hk}$$

We have the following commutatio formula involving cartan's covariant derivative.

$$(1.5)\ \ 2\, T^i_{j|[hk]} = -\,\dot{\partial}_r T^i_j\, K^r_{hk} + T^s_j K^i_{shk} - T^i_s K^s_{jhk}$$

Where

$$(1.6)\ \ K^i_{hjk}(x,\dot{x})\ \underline{\underline{\text{def}}}\ 2\left\{ \partial_{[k}\Gamma^{*i}_{j]h} + \dot{\partial}_s \Gamma^{*i}_{h[j} G^s_{k]} + \Gamma^{*s}_{h[j}\Gamma^{*i}_{k]s} \right\}$$

The commutation formula for a tensor T^i_j will show the role of curvature tensor and torsion tensor as follows.

$$(1.7)\ \ T^i_{l|k|j} - T^i_{l|j|k} = T^h_l R^i_{hkj} - T^i_h R^h_{lkj} - T^i_{l|h} R^h_{kj}$$

where R^i_{hkj} is called cartan third curvature tensor.

$$(1.8)\ \ T^i_{l|k|j} - T^i_{l|j|k} = T^h_l S^i_{hkj} - T^i_h S^h_{lkj}$$

where S^i_{hkj} is called cartan first curvature tensor.

Torse-Forming Vector Field

Definition (2.1) : A vector field vi said to be a torse-forming vector field in Finsler space Fn. If it satisfies the condition.

$$(2.1)\ \ V^i_{|j} = V^i{}_j + \alpha\, \delta^i_j$$

where α is non zero scalar function and ^{j} being any non-null vector field.

The scalar function α appearing in (2.1) is a point function and satisfy following

$$(2.2)\ \ \dot{\partial}_j(\alpha) = 0 \qquad [3]$$

and

(2.3) $\alpha_{|h} = \alpha_h$

Definition 2.2: We consider an infinitesimal transformation of the form.

(2.4) $$\bar{x}^i = x^i + v^i(x)\,dt, \quad v^i_{|j} = v^i{}_j + \alpha\delta^i_j$$

Such a transformation is called a torse-forming infinitesimal transformation.

1. CURVATURE TENSORS FINSLERS-ADMITING TORSE-FORMING VECTOR FIELD

Differentiating (2.1) Covariantly with respect to xk, we get

(3.1) $$V^i_{|j|k} = v^i_{|k}{}_j + v^i{}_{j|k} + \alpha_{|k}\delta^i_j$$

Or

$$V^i_{|j|k} = \left(V^i{}_k + \alpha\delta^i_k\right){}_j + V^i{}_{j|k} + \alpha_{|k}\delta^i_j$$

(3.2) $$V^i_{|j|k} = V^i{}_k{}_j + \alpha{}_j\delta^i_k + V^i{}_{j|k} + \alpha_{|k\,j}\delta^i_j$$

interchanging k and j in equation (2.3), we get

(3.3) $$V^i_{|k|j} = V^i{}_j{}_k + \alpha{}_k\delta^i_j + V^i{}_{k|j} + \alpha_{|j}\delta^i_k$$

Subtracting equation (3.2) and (3.3), we get

(3.4) $$V^i_{|j|k} - V^i_{|k|j} + \alpha\left({}_j\delta^i_k - {}_k\delta^i_j\right) + V^i\left({}_{j|k} - {}_{k|j}\right) + \alpha_{|k}\delta^i_j - \alpha_{|j}\delta^i_k$$

using equation (2.3) in equation (3.4), we get

(3.5) $$V^i_{|j|k} - V^i_{|k|j} = \alpha({}_j\delta^i_k - {}_k\delta^i_j) + V^i({}_{j|k} - {}_{k|j}) + \alpha_k\delta^i_j - \alpha_j\delta^i_k$$

we obtain to the commutation formula (1.5)

(3.6) $$V^i_{|j|k} - V^i_{|k|j} = \dot{\partial}_r V^i K^r_{jk} + V^s K^i_{sjk}$$

In view of (3.5) and (3.6), we get

(3.7) $$\alpha\left({}_{j}\delta^{i}_{k} - {}_{k}\delta^{i}_{j}\right) + v^{i}\left({}_{j|k} - {}_{k|j}\right) + \alpha_{k}\delta^{i}_{j} - \alpha_{j}\delta^{i}_{k} = -\dot{\partial}_{r}V^{i}K^{r}_{jk} + V^{s}K^{i}_{sjk}$$

or

(3.8) $$V^{s}K^{i}_{sjk} = \dot{\partial}_{s}V^{i}K^{r}_{jk} + \alpha\left({}_{j}\delta^{i}_{k} - {}_{k}\delta^{i}_{j}\right) + V^{i}\left({}_{j|k} - {}_{k|j}\right) + \alpha_{k}\delta^{i}_{j} - \alpha_{j}\delta^{i}_{k}$$

or

(3.9) $$V^{s}K^{i}_{sjk} = \dot{\partial}_{r}V^{i}K^{r}_{jk} + \alpha A + {}_{jk}V^{i} + C$$

Where

(a) $$A = {}_{j}\delta^{i}_{k} - {}_{k}\delta^{i}_{j}$$

(b) $${}_{jk} = {}_{j|k} - {}_{k|j}$$

(c) $$C = \alpha_{k}\delta^{i}_{j} - \alpha_{j}\delta^{i}_{k}$$

Thus, we have

THEOREM (3.1) : In a Finsler space Fn, the torse forming transformation (2.1) generators non-flat space.

In view of (3.9b) if ${}_{jk}$ is symmetric, then we have,

(3.10) $${}_{jk} = {}_{j|k} - {}_{k|j} = 0$$

Introducing (3.10) in equation (3.8), we get

(3.11) $$V^{s}K^{i}_{s\,jk} = \dot{\partial}_{r}V^{i}K^{r}_{jk} + \alpha\left({}_{j}\delta^{i}_{k} - {}_{k}\delta^{i}_{j}\right) + \alpha_{k}\delta^{i}_{j} - \alpha_{j}\delta^{i}_{k}$$

Contracting the above equation with respect to the indices i and k, we get

(3.12) $$V^{s}K_{s\,j} = \dot{\partial}_{r}V^{i}K^{r}_{ji} + \alpha\left(n\ {}_{j} - {}_{j}\right) + \alpha_{j} - n\alpha_{j}$$

or

(3.13) $$V^{s}K_{s\,j} = \dot{\partial}_{r}V^{i}K^{r}_{ji} + (n-1)\alpha\ {}_{j} - (n-1)\alpha_{j}$$

(3.14) $$_j = \frac{V^s K_{sj} - \dot{\partial}_r V^i K^r_{ji} + (n-1)\alpha_j}{(n-1)\alpha}$$

Hence we can state :

THEOREM (3.2): In a Finsler space Fn equipped with torse-forming transformation (2.4) in view of cartain curvature tensor K^i_{hjk}, the is non-null vector field ^{j} is given by (3.14) if ^{jk} is symmetric.

Torse Forming Curvature R^i_{hjk} :

Introducing equation (2.4) in (1.7), we get

3.15 $$V^i_{|j|k} - V^i_{|k|j} = V^i_{|h} R^h_{kj} - V^h R^i_{hkj}$$

Using equation (3.5) in (3.15), we get

3.16 $$\alpha\left({}_j \delta^i_k - {}_k \delta^i_j \right) + V^i\left({}_{j|k} - {}_{k|j} \right)$$

$$+ \alpha_k \delta^i_j - \alpha_j \delta^i_k = V^i_{|h} R^h_{kj} - V^h R^i_{hkj}$$

In view of (2.4) and (3.16), we get

3.3 $$\alpha\left({}_j \delta^i_k - {}_k \delta^i_j \right) + V^i\left({}_{j|k} - {}_{k|j} \right)$$

$$+ \alpha_k \delta^i_j - \alpha_j \delta^i_k = \left(V^i \; {}_h + \alpha \delta^i_h\right) R^h_{kj} - V^h R^i_{hkj}$$

3.18 $$\alpha\left({}_j \delta^i_k - {}_k \delta^i_j \right) + V^i\left({}_{j|k} - {}_{k|j} \right)$$

$$+ \alpha_k \delta^i_j - \alpha_j \delta^i_k = \left(V^i \; h + R^i_h\right) R^h_{kj} - V^h R^i_{hkj}$$

3.19 $$V^h R^i_{hkj} = V^i \; {}_h R^h_{kj} + \alpha R^i_{kj} + \alpha\left({}_k \delta^i_j - {}_j \delta^i_k \right)$$
$$+ V^i\left({}_{k|j} - {}_{j|k} \right) + \alpha_j \delta^i_k - \alpha_k \delta^i_j$$

Hence we can state.

Theorem 3.3: In a Finsler space Fn equipped with torse-forming transformation (2.4), the torse-forming vector Vi satisfies the relation (3.19)

Contracting the above equation with respect to the indices i and j, we get

$$3.20 \quad V^h R^i_{hki} = V^i \;{}_h R^h_{ki} + \alpha R^i_{ki} + \alpha\left({}_k \delta^i_i - {}_i \delta^i_k \right) + V^i\left({}_{k|i} - {}_{i|k} \right) + \alpha_i \delta^i_k - \alpha_k \delta^i_i$$

or

$$3.21 \quad V^h R_{hk} = {}_h\left(V^i R^h_{ki}\right) + \alpha R^i_{ki} + \alpha\left(n \;{}_k - {}_k\right) + V^i\left({}_{k|i} - {}_{i|k} \right) + \alpha_k h \alpha_k$$

or

$$3.22 \quad V^h R_{hk} = {}_h\left(V^i R^h_{ki}\right) + \alpha R^i_{ki} + \alpha(n-1) \;{}_k + V^i\left({}_{k|i} - {}_{i|k} \right) + (n-1)\alpha_k$$

$$3.23 \quad V^i\left({}_{k|i} - {}_{i|k} + {}_h R^h_{ki} \right) + \alpha(n-1) \;{}_k + \alpha R^i_{ki} - (n-1)\alpha_k - V^h R_{hk} = 0$$

Thus we have following.

THEOREM 3.4 : In Finsler space Fn equipped with torse-forming transformation (2.4), the non-null vector field $\;^j$ satisfy differential equation (3.23)

Introducing (2.4) in commutation formula (1.8), we get

$$3.24 \quad V^i_{|k|j} - V^i_{|j|k} = V^h S^i_{hkj}$$

or

$$3.25 \quad V^i_{|j|k} - V^i_{|k|j} = -V^h S^i_{hkj}$$

using equation (3.5) in equation (3.25), we get

$$3.26 \quad \alpha\left({}_j \delta^i_k - {}_k \delta^i_j \right) + V^i\left({}_{j|k} - {}_{k|j} \right) + \alpha_k \delta^i_j - \alpha_j \delta^i_k = -V^h S^i_{hkj}$$

$$3.27 \quad V^h S^i_{hkj} = \alpha\left(\;_k \delta^i_j - \;_j \delta^i_k \right) + V^i\left(\;_{k|j} - \;_{j|k} \right) + \alpha_j \delta^i_k - \alpha_k \delta^i_j$$

Hence, we can state:

Theorem 3.5: In Finsler space Fn. The Cartain first curvature tensor S^i_{hkj} with torse-forming transformation (2.4) generates non-flat space.

Contractingi and j in equation (3.27)

$$3.28 \quad V^h S^i_{hki} = \alpha\left(\;_k \delta^i_i - \;_i \delta^i_k \right) + V^i\left(\;_{k|i} - \;_{i|k} \right) + \alpha_i \delta^i_k - \alpha_k \delta^i_i$$

or

$$3.29 \quad V^h S_{shk} + \alpha\left(h \;_k - \;_k\right) + V^i\left(\;_{k|i} - \;_{i|k} \right) + \alpha_k - n\,\alpha_k$$

$$3.30 \quad V^h S_{hk} + (n-1)\alpha_k - V^i\left(\;_{k|i} - \;_{i|k} \right) - \alpha\left(n \;_k - \;_k\right) = 0$$

$$3.31 \quad V^h S_{hk} - V^i\left(\;_{k|i} - \;_{i|k} \right) - \alpha(n-1) \;_k + (n-1)\alpha_k = 0$$

Hence we can state.

THEOREM 3.6 : In Finsler space Fn equipped with torse-forming transformation (2.4) the non-null vector field $\;^j$ satisfy the relation (3.31).

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