Modelling and Forecasting Packaged Food Product Sales Using Mathematical Programming

Saurabh Gupta[1], Nishant Kumar[2]

saurabhgupta2dams@gmail.com

Abstract

Sales forecasting is one of the most common phenomena observed in industry, as it assists other subsidiary department of the industry such as finance, human resources, marketing, supply chain etc. Although forecasted values are obtained through several qualitative and quantitative methods, each method has its own pros and cons. The selection of these models depends upon the knowledge, availability of data and context of forecasting. The purpose of this research paper is to forecast the packaged food product sales using mathematical programming. The scope of the subject is wide and the techniques chosen reflect particular interests and concerns.

Methodology: In this study Linear Programming is used to estimate the parameters of time series forecast by minimising one error index (MAD, MAPE, MPE) and they are compared with the time series forecasting method.

Findings: Linear Programming is used to estimate the parameters of times series forecast with optimisation objectives to minimise forecasting error and it is compared them with the traditional time series forecasting models. The linear programming approach improves the accuracy of forecast and outperforms all the other techniques.

Practical Implications: The use of a mathematical programming provides a formal, logical way of thinking about this decision process. This should increase the understanding of this problem area and increase the quality of decisions.

Keywords

Sales Forecasting, Time Series Analysis, Forecast accuracy, Mathematical Programming.

e-ISSN: 2321-1814, ISBN (Print): 978-1-62951-946-3

Introduction

Due to the strong competition that exists today, most manufacturing organizations give more consideration on continuous effort for increasing their profits and reducing their costs. Accurate sales forecasting is certainly helps the organization to meet the aforementioned goals, since this leads to improved customer service, reduced lost sales and product returns and more efficient production planning.

A perfect forecast is usually impossible because too many factors in the business environment cannot be predicted with certainty. Therefore, rather than search for the perfect forecast, it is far more important to establish the practice of continual review of forecasts and to learn to live with inaccurate forecasts. Because forecasts based on past data, it is not possible to predict the future cent percent accurate. Forecast accuracy decreases as time horizon increases. The accuracy of the forecast and its costs are interrelated. In general, the higher the need for accuracy translates to higher costs of developing forecasting models.

The products which show seasonal fluctuation due to the essence of its use on particular point of time are known as seasonal products. Some examples of seasonal food products are: soft drinks, ice-creams, energy drinks, mosquito repellent, coffee, etc. The big names in the food segments include Nestle, Amul, Unilever, Mother dairy, Coca-Cola, Pepsi, etc. As we know that these seasonal products have high demand in the peak period and low demand/ no demand in the off period, that's why it is very necessary to forecast accurately the future expected sale of that kind of products in order to fulfil the demand of customers as well as reducing cost of inventory handling and customers' lost.

Food companies who offered packaged products are more concerned with sales forecasting due to their special characteristics, such as the short shelf-life of their products, they need to maintain high product quality and the uncertainty and fluctuations in consumer demands. As products can only be sold for a limited period of time, both shortage and surplus of goods can lead to loss of income for the company. The variations in consumer demand are caused by many factors other than climate changes like price, promotions, competitor's activity, consumer's income, changing consumer preferences etc.

Review of Literature

The methodologies that have been used in sales forecasting are typically time series algorithms that can be classified as linear or nonlinear, depending on the nature of the model they are based on. Linear models, like simple and double moving average, single exponential smoothing, holt exponential smoothing, winter's exponential smoothing, decomposition method, autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) (Box, Jenkins, &Reinsel, 1994) are the popular methodologies, but their forecasting ability is limited by their assumption of a linear behaviour and thus, it is not always satisfactory (Zhang, 2003). In order to address possible nonlinearities in time series modelling, researchers introduced a number of nonlinear methodologies, including nonlinear ARMA time series models. Their main drawback is that the type of nonlinearity is not known in advance and the modeller needs to select the structure of the model by trial and error.

Obviously, the key question concerns the accuracy of each modelling method. To this end, a number of studies have been conducted to compare the aforementioned methods; the results are not clearly in favour of one particular method. Zhang (2003) pointed out that no single method is best in every situation and that combining different models is an effective and efficient way to improve forecasting accuracy.

For forecasting productivity, demand and sales of different products there are ample evidence of applications of various time series techniques. While Ghosh (2008) applied univariate time-series techniques such as Multiplicative Seasonal Autoregressive Integrated Moving Average (MSARIMA) and Holt-Winters Multiplicative Exponential Smoothing (ES) for seasonally unadjusted monthly data spanning from April 2000 to February 2007 to forecast the monthly peak demand of electricity in the northern region of India. Mandal (2005) have applied ARIMA model for forecasting sugarcane productions in India with annual data from 1950-51 to 2002-03.Wankhade et. al. (2010) have considered ARIMA model for forecasting pigeon pea production in India with annual data from 1950-51 to 2007-08. Using univariate time series analysis, Rothman (1999), Johnes (1999), Proietti (2001), Gil-Alana (2001), have forecasted the employment rate in US and UK.

While mathematical programming has been shown to be a strong tool for solving statistical problems, the research in the area of forecasting applications is limited. There are a few studies about the usage of optimisation as a supplementary tool for forecasting. Examples are this of Mosheiov and Raven (1997) who apply simple linear programming to estimate the trend of a time series. Dhahri and Chabchoub (2007) have presented a non-linear goal programming model as a tool to estimate the optimal order of ARIMA models.

Problem Statement

Packaged Food products are such a short life span type of product, the company wishes to have a good forecasting that could be used to support decision making in the operational level of supply chain. The tolerance of excess stocks for this type of product is smaller than for a non-perishable product. Besides the obvious risk of financial lost that involved due to an overstock condition, a problem related to an environmental issue may be raised. Meanwhile, out of stock condition has never been an option in the company's policy for any type of its product at all. Especially in a new development stage, out of stock would harm the product penetration both to the customers and consumers.
However, to have a good forecasting is not an easy task. The uncertainty that is always involved in forecasting, for instance people's behaviour in refrigerated prepared food consumption, competitors' activity, price elasticity, increases the difficulties in forecast these seasonal products.

Objective of the Study

Therefore, the aim of this research is to develop a forecasting concept that is able to provide a realistic prediction of the future and valuable for users since it supports them in decision making, in particular within the supply chain area. Though many forecasting theories have been developed, however, it is realized that every forecasting method has trade-offs. The first trade-off occurs in accuracy of the method and effort to perform it (Makridakis and Wheelwright, 1989). The more sophisticated the method used, it might provide better accuracy, but it might involve more cost. Another important trade-off occurs in model generality. A more detailed model, with additional forecasting parameters, may provide better forecasts in sample. However, these models are usually less able to generalize out of sample, and to new and unexpected circumstances. Therefore, this study should also take into account these trade-offs, so that the result will be realistic to be applied in the company.

Time Series Forecasting Methods

A time series is a set of observations measured sequentially through time. These measurements may be made continuously through time or be taken at a discrete set of time points. By convention, these two types of series are called continuous and discrete time series, respectively, even though the measured variable may be discrete or continuous in either case.

Components of Time Series

The pattern or behaviour of data in a time series has several components:
The trend component accounts for the gradual shifting of the time series to relatively higher or lower values over a long period of time. The seasonal component accounts for regular patterns of variability within certain time periods, such as a year. Any regular pattern of sequences of values above and below the trend line lasting more than one year can be attributed to the cyclical component. The irregular component is caused by short-term, unanticipated and non-recurring factors that affect the values of the time series.
In general, the seasonal factor for any period of a year (a quarter, a month, etc.) measures how that period compares to the overall average for an entire year. Specifically, using historical data, the seasonal factor is calculated to be:
Seasonal factor = average for the period / overall average

Seasonally adjusted value = actual value / seasonal factor

The available traditional time series forecasting approaches are divided into two groups i.e. the univariate time series model and multivariate time series model. One of the major limitations of traditional statistical methods is that they are essentially linear methods.

Simple and Double Moving Average

The method of moving average eliminates randomness by taking a set of observed values, finding their average and then using that average as a forecast for the coming period. The term moving average is used because as each new observation becomes available, a new average can be computed and used as a forecast. To determine the appropriate periods of moving average, it is useful to perform forecast by using different average periods and then compute the forecasts errors of each forecast.
Double Moving Average method calculates a second moving average from the original moving average which has been presented previously as an attempt to eliminate systematic error. Simple and Double Moving average methods are appropriate to handle a horizontal data series, but these techniques may be ineffective in handling data series which involves trends and seasonality patterns.

Single Exponential Smoothing

A strong argument can be made that since the most recent observations contain the most current information about what will happen in the future, thus they should be given relatively more weight than the older observations. Exponential smoothing satisfies this requirement and eliminates the need for storing the historical values of the variable likewise in the moving average method. This method is appropriate in handling data series that contains a horizontal pattern. However, this technique may not be effective in handling trends and seasonal patterns.

Linear (Holt's) Exponential Smoothing

If a single exponential smoothing is used with a data series that contains a consistent trend, the forecast will trail behind (lag) that trend. In this case, the linear (Holt's) Exponential smoothing performs well in handling a consistent trend in data series.

Winter's Linear and Seasonal Exponential Smoothing

The winter's Exponential Smoothing involves three parameters. Besides parameters for smoothing the series and trend factor that have been mentioned in the Holt's exponential smoothing, another parameter incorporated in this model is a parameter to smooth the seasonality index.

ARIMA (Auto Regressive, Integrated and Moving Average)

ARIMA model, which also called Box-Jenkins method, has three components that are auto regressive, integrated and moving average. The purpose of ARIMA is to find a model that accurately represents the past and future patterns of a time series where the pattern can be random, seasonal, trend, cyclical, promotional or a combination of patterns until the errors are distributed as white noise. ARIMA models resemble other univariate forecasting method because they include trend, seasonal, and random components.

Mathematical models for Management

A business firm must make decisions on many issues relating to production and inventory planning, sales forecasting, capital budgeting, investment planning, materials requirement planning, personnel management and planning, pricing, distribution, management and planning, integrated supply chain or logistics management and planning, as they affect the several parts of the firm. Business decision making requires the choice of the best decision among alternatives, or at any rate a decision that gives a substantial improvement. The objectives may be revenue maximization, cost minimization, satisfactory performance regarding social responsibility, shareholder value maximization, etc., or sometimes the survival of the firm in adverse circumstances. Methods of Operations Research (O.R.) are well adapted to such decision making in business. In Operations Research, we set up and use mathematical models, usually related to

e-ISSN: 2321-1814, ISBN (Print): 978-1-62951-946-3

questions of planning in business, industry, or management. Any model of a real-life situation must simplify it greatly, by picking out those factors we think important for our purpose, and neglecting the rest. The model results provide useful information for making business decisions.

Research Methodology

The purpose of this research study is to find a suitable model for packaged food products to forecast the expected future sale of goods. Various univariate time series forecasting models for forecasting seasonal food product sales have been applied in this paper. It compares the out-of-sample forecast accuracy of different models using mean absolute deviation, mean absolute percentage error, and mean percentage error.

This study can be best classified as an explorative research design, because it test the implementation of an idea that already exists. A methodology using linear programming is used to forecast sales of packaged food products. This research aims to test the performance of LP-based approaches as an alternative and also to use it for solving problems that traditional statistical tools cannot deal with. The study is mainly focused on this specific ARIMA models due to the limitations of the LP. Only autoregressive models can be formulated as linear programs and the above minimisation objectives can follow a linear structure.

Findings and Analysis

For the requirement of this study two basic methods are used. The first is to forecast the packaged food product sales with the help of time series sales forecasting models. The second is to use a linear programming strategy to predict the sales of packaged food products. In this research paper the univariate Time series Forecasting Methods have been used for computation with the help of past years sales data. There are some other factors also which affect the sale of the product. In order to identify the most accurate forecast method, ex post indicators is used. The main ex post indicators defining the forecast accuracy are as follows (Boguslauskas, 2007):

$$\text{Residual error } (e_t) = Y_t - \hat{Y}_t$$

Where Y_t = Actual value, $\hat{Y}_t$ = Forecast value

1. Mean Absolute Deviation (MAD) demonstrates the absolute residual error and is calculated as follows:

$$MAD = \frac{1}{n}\sum_{t=1}^{n} |Y_t - \hat{Y}_t|$$

2. Mean absolute percentage error (MAPE) that reflects the relative forecast accuracy, is calculated as follows:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} |Y_t - \hat{Y}_t| / Y_t \times 100\%$$

e-ISSN: 2321-1814, ISBN (Print): 978-1-62951-946-3

3. Mean percentage error (MPE) demonstrates the forecast deviation and is calculated in the following way:

$$MPE = \frac{1}{n}\sum_{t=1} (Yt - \hat{Y}t) / Yt \times 100\%$$

Mathematical Programming for Forecasting: The linear programming formulation for the estimation of MinMAD, MinMAPE&MinMPE is as follows:

$$\text{Min}\sum_{t-1}^{n} \boldsymbol{e1}\text{t} - \sum_{t=1}^{n} \boldsymbol{e2t} \text{ [Objective function]}$$

Where e1i is the under-estimation and e2i the over-estimation error; thus, the total estimation error is ei = e1i – e2i. According to this, the constraints of the model are:

$$\sum_{t=1}^{n} [\hat{Y}t] + e1t - e2t = Yt$$

For t = 1, 2, 3........n

e1t and e2t , non-negative.

Forecast percentage error estimate results

Table 1

		Type of error		
Forecasting Method	Product	MAD	MAPE	MPE
Simple Moving Average Method where k=3	Tata Tea	9,106,087.72	0.1575	0.133
Double moving average where k=3		14,367,918.13	0.244	0.212
Exponential Smoothing Method where α=0.1		13,885,578.44	0.175	0.134
Holt Exponential smoothing where α=0.1 and β=0.3		8,550,854.34	0.1176	0.0659
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		11,955,171.61	0.1557	0.1096
ARIMA Method		6,741,906.20	0.1164	0.042
LP Approach		5,626,716.63	0.0021	0.0022
Simple Moving Average Method where k=3	HUL Coffee	3,559.18	0.21	0.1793
Double moving average where k=3		5,651.98	0.3199	0.28
Exponential Smoothing Method where α=0.1		5,355.40	0.228	0.107

Holt Exponential smoothing where α=0.1 and β=0.3		5,372.85	0.2163	-0.172
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		5,419.87	0.2233	-0.07
ARIMA Method		2,443.09	0.134	0.078
LP Approach		1,425.82	0.012	0.0145
Simple Moving Average Method where k=3	Cadbury Chocolates	793,636.32	0.69	-0.377
Double moving average where k=3		1,267,206.73	1.553	-1.14
Exponential Smoothing Method where α=0.1		1,532,324.99	3.4311	-3.34
Holt Exponential smoothing where α=0.1 and β=0.3		2,145,101.07	3.594	3.159
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		2,032,992.93	3.963	1.455
ARIMA Method		487,388.27	0.526	0.037
LP Approach		242,526.01	0.023	0.013
Simple Moving Average Method where k=3	DaburChywanprash	1,146.92	0.164	0.16
Double moving average where k=3		1,912.10	0.249	0.249
Exponential Smoothing Method where α=0.1		3,391.32	0.311	0.19
Holt Exponential smoothing where α=0.1 and β=0.3		1,585.12	0.148	-0.128
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		1,455.72	0.165	-0.051
ARIMA Method		618.00	0.09	0.052
LP Approach		312.82	0.0013	0.00013
Simple Moving Average Method where k=3	Haldiram Processed Food	1,284.52	0.283	0.283
Double moving average where k=3		2,124.96	0.4313	0.4313
Exponential Smoothing Method where α=0.1		1,165.44	0.179	0.031
Holt Exponential smoothing where α=0.1 and β=0.3		179.62	0.031	-0.005

Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		676.14	0.1153	0.075
ARIMA Method		552.07	0.132	0.091
LP Approach		212.828	0.015	0.0142
Simple Moving Average Method where k=3	ITC Packaged Food	158,636.87	0.364	0.364
Double moving average where k=3		272,593.88	0.5302	0.5302
Exponential Smoothing Method where α=0.1		373,289.49	0.8129	-0.164
Holt Exponential smoothing where α=0.1 and β=0.3		86,760.24	0.105	-0.005
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		244,929.51	0.666	-0.175
ARIMA Method		62,120.54	0.2	0.133
LP Approach		43,627.81	0.152	0.128
Simple Moving Average Method where k=3	Pepsi Agro Foods	3,111.43	0.227	0.005
Double moving average where k=3		3,937.93	0.363	0.131
Exponential Smoothing Method where α=0.1		3,138.40	0.356	-0.212
Holt Exponential smoothing where α=0.1 and β=0.3		3,160.81	0.352	-0.117
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		3,143.11	0.359	-0.188
ARIMA Method		3,383.80	0.385	-0.07
LP Approach		2,627.82	0.1287	0.012
Simple Moving Average Method where k=3	Britannia Bread	11,690.63	0.163	0.1231
Double moving average where k=3		19,552.59	0.255	0.1859
Exponential Smoothing Method where α=0.1		23,016.49	0.243	0.049
Holt Exponential smoothing where α=0.1 and β=0.3		25,705.29	0.303	0.003
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		29,553.00	0.344	-0.002
ARIMA Method		6,457.00	0.118	0.025
LP Approach		5,828.11	0.0182	0.0124

e-ISSN: 2321-1814, ISBN (Print): 978-1-62951-946-3

Simple Moving Average Method where k=3	Nestle Milk Powder	8,249.59	0.154	0.143
Double moving average where k=3		14,030.80	0.245	0.2412
Exponential Smoothing Method where α=0.1		25,209.03	0.262	0.248
Holt Exponential smoothing where α=0.1 and β=0.3		15,994.90	0.161	0.1447
Winter's Exponential smoothing(α=0.1,β=0.3,γ=0.5)		22,656.70	0.223	0.214
ARIMA Method		6,823.24	0.1223	0.1223
LP Approach		4,271.52	0.0172	0.01826

From the analysis of the given data set it is found that the Linear Programming Method is best suitable model for forecasting the sales of seasonal food products. The result have shown that the values of forecast error, MAPE, MPE for the period 2004-2013 for most of the products are lowest in ARIMA Method.

Conclusion

Seasonal products are such a short life span type of product. The tolerance of excess stocks for this type of product is smaller than for a non-perishable product. These food products show frequent changes in sales in different seasons. Therefore it is very necessary to forecast the sale of these products accurately. Forecasting accurate seasonal food products sales is extremely complicated and requires lots of coordination from both IT systems and human interaction.

But with the help of the use of mathematical programming it is possible to forecast the sales for seasonal food products accurately. Linear programming is not viewed here as only a mathematical algorithm but as a cognitive process of decision making. The linear programming methodology looks at deviations from specified goals (i.e. minimizing the different types of error). This is very similar to specifying minimum and/or maximum acceptable levels on dimensions depending on how the deviations are minimized.

References

[1] Armstrong, J. Scott (2001), Principles of Forecasting, a handbook for researchers and practitioners, Kluwer Academic Publisher, USA.

[2] Box, G.E.P., Jenkins, G.M &Reinsel, G.C. (1994), Time series analysis: Forecasting and control (3rd Ed.) Englewood cliffs, NJ: Prentice Hall.

[3] Charnes A., & W.W. Cooper (1961), "Management Models and Industrial Applications of Linear Programming," New York: John Wiley & Sons.

[4] Danatzig, George B. (1948), “Programming in a Linear Structure,” Comptroller, USAF, Washington D.C.

[5] Dhahri I. and H. Chabchoub (2007). Nonlinear goal programming models quantifying the bullwhip effect in supply chain based on ARIMA parameters. European Journal of Operational Research, 177(3), pp. 1800 – 1810.

[6] Fildes, R., Hibon, M., Makridakis, S., Meade, N., 1998. Generalising about univariate forecasting methods: Further empirical evidence, International Journal of Forecasting 14, 339-358.

[7] Gardner, E.S., Jr., McKenzie, E., 1985. Forecasting trends in time series, Management Science 31, 1237-1246.

[8] Ghosh, S, 2008. "Univariate Time-Series Forecasting of Monthly Peak Demand of Electricity in Northern India". International Journal of Indian Culture and Business Management, 1(4); 466 - 474.

[9] Harvey, A.C., 1990. Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, New York.

[10] Johnes, G.1999,"Forecasting Unemployment". Applied Financial Economics, 6, 605-607.

[11] Lundholm, Russell J. and McVay, Sarah E, “Forecasting Sales: A model and some evidence from the retail industry.” Jan 2004.

[12] Makridakis, S. Forecasting: ‘Methods and Applications’, 3rd ed. / S. Makridakis, S.C. Wheelwright, R. J. Hyndman, New York: John Wiley & Sons, 1998.

[13] Mandal, B. N., 2005. "Forecasting Sugarcane Productions in India with ARIMA Model".

[14] Mosheiov G. and A. Raveh (1997). On trend estimation of time-series: a simple linear programming approach. Journal of the Operational Research Society, 48(1), pp. 90 – 96.